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Topological Recursion Relations on $\overline{\mathcal{M}}_{3,2}$

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Abstract

In this paper, we give some new genus-3 universal equations for Gromov-Witten invariants of compact symplectic manifolds. These equations were obtained by studying new relations in the tautological ring of the moduli space of 2-pointed genus-3 stable curves. A byproduct of our search for genus-3 equations is a new genus-2 universal equation for Gromov-Witten invariants.

It is well known that relations in the tautological ring of moduli space of stable curves $\overline{\mathcal{M}}_{g,n}$ produce universal equations for Gromov-Witten invariants of all compact symplectic manifolds. A typical genus-0 example is the associativity equation for the quantum cohomology, also known as the WDVV equation. Finding explicit higher genus universal equations is a very difficult problem. Genus-1 and genus-2 universal equations were discovered in [Ge1], [Ge2], and [BP]. Relations among these equations were studied in [L2]. For manifolds with semisimple quantum cohomology, these equations determine the genus-1 and genus-2 Gromov-Witten invariants in terms of genus-0 invariants (cf. [DZ] for the genus-1 case and [L1] for the genus-2 case). In [KL], the authors proved a genus-3 topological recursion relation by studying a tautological relation on $\overline{\mathcal{M}}_{3,1}$. Certain topological recursion relations of all genera were proved in [LP]. Despite all these progresses, the understanding of universal equations is still very limited and unsatisfactory. For example, so far the known genus-3 equations still can not determine the genus-3 generating function even for manifolds with semisimple quantum cohomology. Therefore it is very interesting to find more genus-3 universal equations. The main purpose of this paper is to obtain new genus-3 equations by studying tautological relations on $\overline{\mathcal{M}}_{3,2}$.

To describe the universal equations, we need to use some operators introduced in [L1]. Let M be a compact symplectic manifold. The *big phase space* for Gromov-Witten invariants of M is a product of infinitely many copies of $H^*(M;\mathbb{C})$. We will choose a basis $\{\gamma_\alpha \mid \alpha = 1, \ldots, N\}$ of $H^*(M;\mathbb{C})$. The quantum product $\mathcal{W}_1 \circ \mathcal{W}_2$ of two vector fields $\mathcal{W}_1$ and $\mathcal{W}_2$ on the big phase space was introduced in [L1]. This is an associative product without an identity element. An operator T on the space of vector fields on the big phase space was also introduced in [L1] to measure the failure of the string vector field to be an identity element with respect to this product. This operator turns out to be a very useful device to translate relations in the tautological rings of $\overline{\mathcal{M}}_{g,n}$ into universal equations for Gromov-Witten invariants. We will write universal equations of Gromov-Witten invariants as equations among tensors $\langle\!\langle \mathcal{W}_1 \cdots \mathcal{W}_k \rangle\!\rangle_g$ which are defined to be the k-th covariant derivatives of the generating functions of genus-g Gromov-Witten invariants

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with respect to the trivial connection on the big phase space. We will briefly review these definitions in Section 1 for completeness.

Define

$$
\begin{aligned}
& Q(\mathcal{W}_1, \mathcal{W}_2) \\
:= \;& \frac{2}{9} \langle\langle \mathcal{W}_2\, T(\gamma^\alpha) \rangle\rangle_2 \left\langle\left\langle \gamma_\alpha\, \mathcal{W}_1\, \gamma^\beta\, \gamma_\beta \right\rangle\right\rangle_0 + \frac{5}{24} \langle\langle T(\mathcal{W}_2)\, \gamma^\alpha \rangle\rangle_2 \left\langle\left\langle \gamma_\alpha\, \mathcal{W}_1\, \gamma^\beta\, \gamma_\beta \right\rangle\right\rangle_0 \\
& + \frac{16}{3} \langle\langle \mathcal{W}_2\, T(\gamma^\alpha) \rangle\rangle_2 \langle\langle \{\gamma_\alpha \circ \mathcal{W}_1\} \rangle\rangle_1 + 5 \langle\langle T(\mathcal{W}_2)\, \gamma^\alpha \rangle\rangle_2 \langle\langle \{\gamma_\alpha \circ \mathcal{W}_1\} \rangle\rangle_1 \\
& + \frac{40}{3} \langle\langle T(\gamma^\alpha) \rangle\rangle_2 \langle\langle \{\gamma_\alpha \circ \mathcal{W}_1\}\ \mathcal{W}_2 \rangle\rangle_1 + \frac{1}{6} \langle\langle \mathcal{W}_2\, T(\gamma^\alpha)\ \{\gamma_\alpha \circ \mathcal{W}_1\} \rangle\rangle_2 \\
& + \frac{1}{2} \langle\langle T(\mathcal{W}_2)\, \gamma^\alpha\ \{\gamma_\alpha \circ \mathcal{W}_1\} \rangle\rangle_2 + \frac{2}{9} \langle\langle \mathcal{W}_1\ \{\mathcal{W}_2 \circ \gamma^\alpha \circ \gamma_\alpha\} \rangle\rangle_2 \\
& + \frac{1}{18} \langle\langle \mathcal{W}_1\, \gamma^\alpha \rangle\rangle_1 \left\langle\left\langle \gamma_\alpha\, \mathcal{W}_2 \left\{\gamma^\beta \circ \gamma_\beta\right\} \right\rangle\right\rangle_1 + \frac{1}{18} \langle\langle \mathcal{W}_1\, \gamma^\alpha \rangle\rangle_1 \left\langle\left\langle \gamma_\alpha\, \gamma^\beta \right\rangle\right\rangle_1 \langle\langle \gamma_\beta\, \mathcal{W}_2\, \gamma^\mu\, \gamma_\mu \rangle\rangle_0 \\
& + \frac{1}{30} \langle\langle \gamma^\alpha \rangle\rangle_1 \left\langle\left\langle \gamma_\alpha\, \mathcal{W}_2\, \gamma^\beta \right\rangle\right\rangle_1 \langle\langle \gamma_\beta\, \mathcal{W}_1\, \gamma^\mu\, \gamma_\mu \rangle\rangle_0 + \frac{1}{30} \langle\langle \gamma^\alpha \rangle\rangle_1 \left\langle\left\langle \gamma_\alpha\, \mathcal{W}_2\, \gamma^\beta\ \{\gamma_\beta \circ \mathcal{W}_1\} \right\rangle\right\rangle_1 \\
& + \frac{9}{10} \left\langle\left\langle \mathcal{W}_2\, \gamma^\alpha\, \gamma^\beta \right\rangle\right\rangle_1 \langle\langle \gamma_\alpha\ \{\gamma_\beta \circ \mathcal{W}_1\} \rangle\rangle_1 + \frac{1}{30} \left\langle\left\langle \gamma^\alpha\, \gamma_\alpha\, \mathcal{W}_2\, \gamma^\beta \right\rangle\right\rangle_1 \langle\langle \{\gamma_\beta \circ \mathcal{W}_1\} \rangle\rangle_1 \\
& + \frac{2}{15} \left\langle\left\langle \gamma^\alpha\, \gamma_\alpha\, \gamma^\beta \right\rangle\right\rangle_1 \langle\langle \{\gamma_\beta \circ \mathcal{W}_2\}\ \mathcal{W}_1 \rangle\rangle_1 + \frac{3}{5} \left\langle\left\langle \mathcal{W}_2\, \gamma^\alpha\, \gamma^\beta \right\rangle\right\rangle_1 \langle\langle \gamma_\alpha\, \gamma_\beta\, \mathcal{W}_1\, \gamma^\mu \rangle\rangle_0 \langle\langle \gamma_\mu \rangle\rangle_1 \\
& + \frac{1}{6} \left\langle\left\langle \mathcal{W}_1\, \gamma^\alpha\, \gamma^\beta \right\rangle\right\rangle_1 \langle\langle \{\gamma_\alpha \circ \gamma_\beta\}\ \mathcal{W}_2 \rangle\rangle_1 + \frac{16}{15} \left\langle\left\langle \gamma^\alpha\, \gamma^\beta \right\rangle\right\rangle_1 \langle\langle \gamma_\alpha\, \gamma_\beta\, \mathcal{W}_1\, \gamma^\mu \rangle\rangle_0 \langle\langle \gamma_\mu\, \mathcal{W}_2 \rangle\rangle_1 \\
& + \frac{41}{90} \langle\langle \mathcal{W}_2\, \gamma^\alpha \rangle\rangle_1 \left\langle\left\langle \gamma_\alpha\, \mathcal{W}_1\, \gamma^\beta\, \gamma_\beta\, \gamma^\mu \right\rangle\right\rangle_0 \langle\langle \gamma_\mu \rangle\rangle_1 + \frac{4}{5} \langle\langle \gamma^\alpha \rangle\rangle_1 \left\langle\left\langle \gamma_\alpha\, \mathcal{W}_2\, \gamma^\beta \right\rangle\right\rangle_1 \langle\langle \{\gamma_\beta \circ \mathcal{W}_1\} \rangle\rangle_1 \\
& + \frac{16}{5} \langle\langle \gamma^\alpha \rangle\rangle_1 \left\langle\left\langle \gamma_\alpha\, \gamma^\beta \right\rangle\right\rangle_1 \langle\langle \{\gamma_\beta \circ \mathcal{W}_2\}\ \mathcal{W}_1 \rangle\rangle_1 + \frac{92}{15} \langle\langle \mathcal{W}_2\, \gamma^\alpha \rangle\rangle_1 \left\langle\left\langle \gamma^\beta \right\rangle\right\rangle_1 \langle\langle \gamma^\mu \rangle\rangle_1 \langle\langle \gamma_\alpha\, \gamma_\beta\, \gamma_\mu\, \mathcal{W}_1 \rangle\rangle_0 \\
& + \frac{1}{720} \left\langle\left\langle \mathcal{W}_2\, \gamma^\alpha\, \gamma_\alpha\, \gamma^\beta \right\rangle\right\rangle_1 \langle\langle \gamma_\beta\, \mathcal{W}_1\, \gamma^\mu\, \gamma_\mu \rangle\rangle_0 + \frac{1}{720} \left\langle\left\langle \gamma^\alpha\, \gamma_\alpha\, \mathcal{W}_2\, \gamma^\beta\ \{\gamma_\beta \circ \mathcal{W}_1\} \right\rangle\right\rangle_1 \\
& + \frac{1}{135} \left\langle\left\langle \mathcal{W}_2\, \gamma^\alpha\, \gamma^\beta\, \gamma^\mu \right\rangle\right\rangle_1 \langle\langle \gamma_\alpha\, \gamma_\beta\, \gamma_\mu\, \mathcal{W}_1 \rangle\rangle_0 + \frac{1}{40} \left\langle\left\langle \mathcal{W}_2\, \gamma^\alpha\, \gamma^\beta \right\rangle\right\rangle_1 \langle\langle \gamma_\alpha\, \gamma_\beta\, \mathcal{W}_1\, \gamma^\mu\, \gamma_\mu \rangle\rangle_0 \\
& + \frac{1}{120} \langle\langle \mathcal{W}_2\, \gamma^\alpha \rangle\rangle_1 \left\langle\left\langle \gamma_\alpha\, \mathcal{W}_1\, \gamma^\beta\, \gamma_\beta\, \gamma^\mu\, \gamma_\mu \right\rangle\right\rangle_0
\end{aligned}
\tag{1}
$$

Note that $Q(\mathcal{W}_1, \mathcal{W}_2)$ only involves data with genus ≤ 2. In this paper we will prove the following genus-3 universal equation

Theorem 0.1 *For all compact symplectic manifolds,*

$$\left\langle\left\langle T^2(\mathcal{W}_1) T(\mathcal{W}_2) \right\rangle\right\rangle_3 - \left\langle\left\langle T^2(\mathcal{W}_2) T(\mathcal{W}_1) \right\rangle\right\rangle_3 = \frac{1}{7} \{Q(\mathcal{W}_1, \mathcal{W}_2) - Q(\mathcal{W}_2, \mathcal{W}_1)\}$$

for all vector fields $\mathcal{W}_1$ and $\mathcal{W}_2$ on the big phase space.

Note that in general

$$\left\langle\left\langle T^2(\mathcal{W}_1) T(\mathcal{W}_2) \right\rangle\right\rangle_3 \neq \frac{1}{7} Q(\mathcal{W}_1, \mathcal{W}_2).$$

Instead $\left\langle\left\langle T^2(\mathcal{W}_1) T(\mathcal{W}_2) \right\rangle\right\rangle_3$ is given by a much more complicated genus-3 universal equation in Theorem 2.2, which is the main result of this paper. Theorem 0.1 is actually a corollary of

Theorem 2.2. We would like to point out that the genus-3 topological recursion relation in [KL] also follows from the formula in Theorem 2.2 (see the remark at the end of Section 2) .

During the proof of Theorem 2.2, we also discovered a new genus-2 universal equation under the assumption that the tautological ring of $\overline{\mathcal{M}}_{3,2}$ is Gorenstein. This genus-2 equation is given in Proposition 2.3. This equation does not seem to follow from known genus-2 equations. By a result of [L1], known genus-2 equations already completely determine the genus-2 generating function for manifolds with semisimple quantum cohomology. Because of this fact, the new genus-2 equation in Proposition 2.3 is quite a surprise.

We also notice that universal equations in this paper correspond to relations in the tautological ring of $\overline{\mathcal{M}}_{3,2}$. We will give the relation corresponding to Theorem 0.1 in Section 3.

The second author would like to thank R. Pandharipande for helpful discussions on whether the tautological ring $R^*(\overline{\mathcal{M}}_{3,2})$ is Gorenstein.

1 Preliminaries

Let M be a compact symplectic manifold. The *big phase space* is by definition the infinite product

$$P := \prod_{n=0}^{\infty} H^*(M;\mathbb{C}).$$

Fix a basis $\{\gamma_0, \ldots, \gamma_N\}$ of $H^*(M;\mathbb{C})$, where γ_0 is the identity element, of the ordinary cohomology ring of M. Then we denote the corresponding basis for the n-th copy of $H^*(M;\mathbb{C})$ in P by $\{\tau_n(\gamma_0), \ldots, \tau_n(\gamma_N)\}$. We call $\tau_n(\gamma_\alpha)$ a *descendant of* γ_α *with descendant level* n. We can think of P as an infinite dimensional vector space with a basis $\{\tau_n(\gamma_\alpha) \mid 0 \leq \alpha \leq N,\ n \in \mathbb{Z}_{\geq 0}\}$ where $\mathbb{Z}_{\geq 0} = \{n \in \mathbb{Z} \mid n \geq 0\}$. Let $(t_n^\alpha \mid 0 \leq \alpha \leq N,\ n \in \mathbb{Z}_{\geq 0})$ be the corresponding coordinate system on P. For convenience, we identify $\tau_n(\gamma_\alpha)$ with the coordinate vector field $\frac{\partial}{\partial t_n^\alpha}$ on P for $n \geq 0$. If $n < 0$, $\tau_n(\gamma_\alpha)$ is understood to be the 0 vector field. We also abbreviate $\tau_0(\gamma_\alpha)$ by γ_α. We use τ_+ and τ_- to denote the operators which shift the level of descendants by 1, i.e.

$$\tau_\pm\left(\sum_{n,\alpha} f_{n,\alpha}\tau_n(\gamma_\alpha)\right) = \sum_{n,\alpha} f_{n,\alpha}\tau_{n\pm 1}(\gamma_\alpha)$$

where $f_{n,\alpha}$ are functions on the big phase space.

We will adopt the following *notational conventions*: Lower case Greek letters, e.g. α, β, μ, ν, σ,..., etc., will be used to index the cohomology classes on M. These indices run from 0 to N. Lower case English letters, e.g. i, j, k, m, n, ..., etc., will be used to index the level of descendants. These indices run over the set of all non-negative integers, i.e. $\mathbb{Z}_{\geq 0}$. All summations are over the entire ranges of the corresponding indices unless otherwise indicated. Let

$$\eta_{\alpha\beta} = \int_M \gamma_\alpha \cup \gamma_\beta$$

be the intersection form on $H^*(M,\mathbb{C})$. We will use $\eta = (\eta_{\alpha\beta})$ and $\eta^{-1} = (\eta^{\alpha\beta})$ to lower and raise indices. For example,

$$\gamma^\alpha := \eta^{\alpha\beta}\gamma_\beta.$$

Here we are using the summation convention that repeated indices (in this formula, β) should be summed over their entire ranges.

Let

$$\langle\, \tau_{n_1}(\gamma_{\alpha_1})\, \tau_{n_2}(\gamma_{\alpha_2})\, \ldots\, \tau_{n_k}(\gamma_{\alpha_k})\, \rangle_{g,d} := \int_{[\overline{\mathcal{M}}_{g,n}(M;d)]^{\mathrm{virt}}} \bigcup_{i=1}^{k} (\Psi^{n_i} \cup \mathrm{ev_i}^{*}\gamma_{\alpha_i})$$

be the genus-g, degree d, descendant Gromov-Witten invariant associated to $\gamma_{\alpha_1}, \ldots, \gamma_{\alpha_k}$ and nonnegative integers $n_1, \ldots, n_k$ (cf. [W], [RT], [LiT]). Here, $\overline{\mathcal{M}}_{g,k}(M;d)$ is the moduli space of stable maps from genus-g, k-pointed curves to M of degree $d \in H_2(M;\mathbb{Z})$. Ψ_i is the first Chern class of the tautological line bundle over $\overline{\mathcal{M}}_{g,k}(M;d)$ whose geometric fiber over a stable map is the cotangent space of the domain curve at the i-th marked point while $\mathrm{ev_i} : \overline{\mathcal{M}}_{g,n}(M;d) \to M$ is the i-th evaluation map for all $i = 1, \ldots, k$. Finally, $[\overline{\mathcal{M}}_{g,n}(M;d)]^{\mathrm{virt}}$ is the virtual fundamental class. The genus-g generating function is defined to be

$$F_g = \sum_{k\geq 0} \frac{1}{k!} \sum_{\substack{\alpha_1,\ldots,\alpha_k \\ n_1,\ldots,n_k}} t^{\alpha_1}_{n_1} \cdots t^{\alpha_k}_{n_k} \sum_{d\in H_2(V,\mathbb{Z})} q^d \,\langle\, \tau_{n_1}(\gamma_{\alpha_1})\, \tau_{n_2}(\gamma_{\alpha_2})\, \ldots\, \tau_{n_k}(\gamma_{\alpha_k})\, \rangle_{g,d}$$

where q^d belongs to the Novikov ring. This function is understood as a formal power series n the variables $\{\, t^{\alpha}_{n} \,\}$ with coefficients in the Novikov ring.

Introduce a k-tensor $\langle\langle\, \underbrace{\cdots\cdots}_{k}\, \rangle\rangle$ defined by

$$\langle\langle\, \mathcal{W}_1 \mathcal{W}_2 \cdots \mathcal{W}_k \,\rangle\rangle_g \;:= \sum_{m_1,\alpha_1,\ldots,m_k,\alpha_k} f^1_{m_1,\alpha_1} \cdots f^k_{m_k,\alpha_k}\; \frac{\partial^k}{\partial t^{\alpha_1}_{m_1} \partial t^{\alpha_2}_{m_k} \cdots \partial t^{\alpha_k}_{m_k}} F_g,$$

for vector fields $\mathcal{W}_i = \sum_{m,\alpha} f^i_{m,\alpha} \frac{\partial}{\partial t^\alpha_m}$ where $f^i_{m,\alpha}$ are functions on the big phase space. This tensor is called the *k-point (correlation) function*.

For any vector fields $\mathcal{W}_1$ and $\mathcal{W}_2$ on the big phase space, the quantum product of $\mathcal{W}_1$ and $\mathcal{W}_2$ is defined by

$$\mathcal{W}_1 \circ \mathcal{W}_2 := \langle\langle\, \mathcal{W}_1\, \mathcal{W}_2\, \gamma^\alpha \,\rangle\rangle_0\, \gamma_\alpha.$$

Define the vector field

$$T(\mathcal{W}) := \tau_+(\mathcal{W}) - \langle\langle\, \mathcal{W}\, \gamma^\alpha \,\rangle\rangle_0\, \gamma_\alpha$$

for any vector field $\mathcal{W}$. The operator T was introduced in [L1] as a convenient tool in the study of universal equations for Gromov-Witten invariants. Let ψ_i be the first Chern class of the tautological line bundle over $\overline{\mathcal{M}}_{g,k}$ whose geometric fiber over a stable curve is the cotangent space of the curve at the i-th marked point. When we translate a relation in the tautological ring of $\overline{\mathcal{M}}_{g,k}$ into differential equations for generating functions of Gromov-Witten invariants, each ψ class corresponds to the insertion of the operator T. Let ∇ be the trivial flat connection on the big phase space with respect to which $\tau_n(\gamma_\alpha)$ are parallel vector fields for all α and n. Then the covariant derivative of the quantum product satisfies

$$\nabla_{\mathcal{W}_3}(\mathcal{W}_1 \circ \mathcal{W}_2) = (\nabla_{\mathcal{W}_3}\mathcal{W}_1) \circ \mathcal{W}_2 + \mathcal{W}_1 \circ (\nabla_{\mathcal{W}_3}\mathcal{W}_2) + \langle\langle\, \mathcal{W}_1\, \mathcal{W}_2\, \mathcal{W}_3\, \gamma^\alpha \,\rangle\rangle_0\, \gamma_\alpha$$

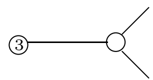
and the covariant derivative of the operator T is given by

$$\nabla_{\mathcal{W}_2}\, T(\mathcal{W}_1) = T(\nabla_{\mathcal{W}_2}\mathcal{W}_1) - \mathcal{W}_2 \circ \mathcal{W}_1$$

for any vector fields $\mathcal{W}_1, \mathcal{W}_2$ and $\mathcal{W}_3$ (cf. [L1, Equation (8) and Lemma 1.5]). We need to use these formulas in order to compute derivatives of universal equations.

2 Topological Recursion Relations on $\overline{\mathcal{M}}_{3,2}$

The cohomology class $\psi_1^2\,\psi_2$ vanishes on $\mathcal{M}_{3,2}$ due to a result of Ionel (cf. [Io]). Furthermore, by a result of Faber and Pandharipande [FP2], $\psi_1^2\,\psi_2$ is equal to a class from the boundary strata which is tautological, and therefore is a linear combination of products of ψ and κ classes and fundamental classes of some boundary strata. For any curve in $\partial\mathcal{M}_{3,2} := \overline{\mathcal{M}}_{3,2} - \mathcal{M}_{3,2}$ with a genus-3 component, it has a dual graph



(see Section 3 for the conventions of dual graphs). Since this graph is a tree, all such curves are of compact type. For this stratum to occur in the expression of $\psi_1^2\,\psi_2$, it must be multiplied by combinations of ψ-classes and κ-classes of degree 2 on the genus-3 component. By a result of Yang [Y], κ-classes in this expression can be replaced with combinations of ψ-classes and fundamental classes of boundary strata. On all other boundary strata, all components of curves must have genus at most 2, therefore κ classes again can be represented as linear combinations of ψ classes and fundamental classes of boundary strata (cf. [AC]). Therefore, it follows that $\psi_1^2\,\psi_2$ on $\overline{\mathcal{M}}_{3,2}$ can be written as a linear combination of products of the ψ classes and the fundamental classes of some boundary strata. By taking into consideration the genus-0 and genus-1 topological recursion relations as well as Mumford's genus-2 relation, we can translate these results into the following universal equations for Gromov-Witten invariants with unknown constants $a_1, \ldots, a_{105}$:

$$
\begin{aligned}
0 \;=\;& \Phi(\mathcal{W}_1, \mathcal{W}_2) \\
:=\;& -\left\langle\!\!\left\langle\, T^2(\mathcal{W}_1) T(\mathcal{W}_2) \,\right\rangle\!\!\right\rangle_3 \\
& + a_1 \left\langle\!\!\left\langle\, T^2(\mathcal{W}_1 \circ \mathcal{W}_2) \,\right\rangle\!\!\right\rangle_3 + a_2 \langle\!\langle\, \mathcal{W}_1\, \mathcal{W}_2\, T(\gamma_\alpha \circ \gamma^\alpha) \,\rangle\!\rangle_2 \\
& + a_3 \langle\!\langle\, T(\mathcal{W}_1)\, \gamma^\alpha \,\rangle\!\rangle_2 \left\langle\!\!\left\langle\, \gamma_\alpha\, \mathcal{W}_2\, \gamma^\beta\, \gamma_\beta \,\right\rangle\!\!\right\rangle_0 + a_4 \langle\!\langle\, \mathcal{W}_1\, T(\gamma^\alpha) \,\rangle\!\rangle_2 \left\langle\!\!\left\langle\, \gamma_\alpha\, \mathcal{W}_2\, \gamma^\beta\, \gamma_\beta \,\right\rangle\!\!\right\rangle_0 \\
& + a_5 \langle\!\langle\, T(\mathcal{W}_2)\, \gamma^\alpha \,\rangle\!\rangle_2 \left\langle\!\!\left\langle\, \gamma_\alpha\, \mathcal{W}_1\, \gamma^\beta\, \gamma_\beta \,\right\rangle\!\!\right\rangle_0 + a_6 \langle\!\langle\, \mathcal{W}_2\, T(\gamma^\alpha) \,\rangle\!\rangle_2 \left\langle\!\!\left\langle\, \gamma_\alpha\, \mathcal{W}_1\, \gamma^\beta\, \gamma_\beta \,\right\rangle\!\!\right\rangle_0 \\
& + a_7 \langle\!\langle\, T(\gamma^\alpha) \,\rangle\!\rangle_2 \left\langle\!\!\left\langle\, \gamma_\alpha\, \mathcal{W}_1\, \mathcal{W}_2\, \gamma^\beta\, \gamma_\beta \,\right\rangle\!\!\right\rangle_0 + a_8 \langle\!\langle\, \mathcal{W}_1\, T(\gamma^\alpha) \,\rangle\!\rangle_2 \langle\!\langle\, \{\gamma_\alpha \circ \mathcal{W}_2\} \,\rangle\!\rangle_1 \\
& + a_9 \langle\!\langle\, T(\mathcal{W}_2)\, \gamma^\alpha \,\rangle\!\rangle_2 \langle\!\langle\, \{\gamma_\alpha \circ \mathcal{W}_1\} \,\rangle\!\rangle_1 + a_{10} \langle\!\langle\, \mathcal{W}_2\, T(\gamma^\alpha) \,\rangle\!\rangle_2 \langle\!\langle\, \{\gamma_\alpha \circ \mathcal{W}_1\} \,\rangle\!\rangle_1 \\
& + a_{11} \langle\!\langle\, T(\gamma^\alpha) \,\rangle\!\rangle_2 \langle\!\langle\, \{\gamma_\alpha \circ \mathcal{W}_1\}\ \mathcal{W}_2 \,\rangle\!\rangle_1 + a_{12} \langle\!\langle\, T(\gamma^\alpha) \,\rangle\!\rangle_2 \langle\!\langle\, \{\gamma_\alpha \circ \mathcal{W}_2\}\ \mathcal{W}_1 \,\rangle\!\rangle_1 \\
& + a_{13} \langle\!\langle\, T(\gamma^\alpha) \,\rangle\!\rangle_2 \left\langle\!\!\left\langle\, \gamma_\alpha\, \mathcal{W}_1\, \mathcal{W}_2\, \gamma^\beta \,\right\rangle\!\!\right\rangle_0 \langle\!\langle\, \gamma_\beta \,\rangle\!\rangle_1 + a_{14} \langle\!\langle\, \mathcal{W}_1\, T(\gamma^\alpha)\ \{\gamma_\alpha \circ \mathcal{W}_2\} \,\rangle\!\rangle_2 \\
& + a_{15} \langle\!\langle\, T(\mathcal{W}_2)\, \gamma^\alpha\ \{\gamma_\alpha \circ \mathcal{W}_1\} \,\rangle\!\rangle_2 + a_{16} \langle\!\langle\, \mathcal{W}_2\, T(\gamma^\alpha)\ \{\gamma_\alpha \circ \mathcal{W}_1\} \,\rangle\!\rangle_2 \\
& + a_{17} \left\langle\!\!\left\langle\, T(\gamma^\alpha)\, \gamma^\beta \,\right\rangle\!\!\right\rangle_2 \langle\!\langle\, \gamma_\alpha\, \gamma_\beta\, \mathcal{W}_1\, \mathcal{W}_2 \,\rangle\!\rangle_0 + a_{18} \langle\!\langle\, T(\gamma^\alpha)\, \gamma_\alpha\ \{\mathcal{W}_1 \circ \mathcal{W}_2\} \,\rangle\!\rangle_2 \\
& + a_{19} \langle\!\langle\, \gamma^\alpha\, \gamma_\alpha\, T(\mathcal{W}_1 \circ \mathcal{W}_2) \,\rangle\!\rangle_2 + a_{20} \langle\!\langle\, T(\gamma^\alpha) \,\rangle\!\rangle_2 \langle\!\langle\, \gamma_\alpha\ \{\mathcal{W}_1 \circ \mathcal{W}_2\} \,\rangle\!\rangle_1 \\
& + a_{21} \langle\!\langle\, T(\mathcal{W}_1 \circ \mathcal{W}_2)\, \gamma^\alpha \,\rangle\!\rangle_2 \langle\!\langle\, \gamma_\alpha \,\rangle\!\rangle_1 + a_{22} \langle\!\langle\, \{\mathcal{W}_1 \circ \mathcal{W}_2\}\ T(\gamma^\alpha) \,\rangle\!\rangle_2 \langle\!\langle\, \gamma_\alpha \,\rangle\!\rangle_1 \\
& + a_{23} \langle\!\langle\, \mathcal{W}_1\ \{\mathcal{W}_2 \circ \gamma^\alpha \circ \gamma_\alpha\} \,\rangle\!\rangle_2 + a_{24} \langle\!\langle\, \mathcal{W}_2\ \{\mathcal{W}_1 \circ \gamma^\alpha \circ \gamma_\alpha\} \,\rangle\!\rangle_2 \\
& + a_{25} \langle\!\langle\, \{\mathcal{W}_1 \circ \mathcal{W}_2\}\ \{\gamma^\alpha \circ \gamma_\alpha\} \,\rangle\!\rangle_2 + a_{26} \langle\!\langle\, \gamma^\alpha \,\rangle\!\rangle_2 \left\langle\!\!\left\langle\, \gamma_\alpha\ \{\mathcal{W}_1 \circ \mathcal{W}_2\}\ \gamma^\beta\, \gamma_\beta \,\right\rangle\!\!\right\rangle_0
\end{aligned}
$$

$$+a_{27}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_2\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\mathcal{W}_1\,\gamma^\beta\,\{\gamma_\beta\circ\mathcal{W}_2\}\,\Big\rangle\!\!\Big\rangle_0+a_{28}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_2\,\Big\langle\!\!\Big\langle\,\Big\{\gamma_\alpha\circ\gamma^\beta\Big\}\,\gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2\,\Big\rangle\!\!\Big\rangle_0$$

$$+a_{29}\,\langle\!\langle\,\gamma^\alpha\,\{\gamma_\alpha\circ\mathcal{W}_1\,\circ\mathcal{W}_2\}\,\rangle\!\rangle_2+a_{30}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_2\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\mathcal{W}_1\,\mathcal{W}_2\,\Big\{\gamma^\beta\circ\gamma_\beta\Big\}\,\Big\rangle\!\!\Big\rangle_0$$

$$+a_{31}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_2\,\langle\!\langle\,\{\gamma_\alpha\circ\mathcal{W}_1\circ\mathcal{W}_2\}\,\rangle\!\rangle_1+a_{32}\,\langle\!\langle\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\Big\{\gamma^\beta\circ\gamma_\beta\Big\}\,\Big\rangle\!\!\Big\rangle_1$$

$$+a_{33}\,\langle\!\langle\,\mathcal{W}_1\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\mathcal{W}_2\,\Big\{\gamma^\beta\circ\gamma_\beta\Big\}\,\Big\rangle\!\!\Big\rangle_1+a_{34}\,\langle\!\langle\,\mathcal{W}_2\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\mathcal{W}_1\,\Big\{\gamma^\beta\circ\gamma_\beta\Big\}\,\Big\rangle\!\!\Big\rangle_1$$

$$+a_{35}\,\langle\!\langle\,\mathcal{W}_1\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\gamma_\beta\,\mathcal{W}_2\,\gamma^\mu\,\gamma_\mu\,\rangle\!\rangle_0+a_{36}\,\langle\!\langle\,\mathcal{W}_2\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\gamma_\beta\,\mathcal{W}_1\,\gamma^\mu\,\gamma_\mu\,\rangle\!\rangle_0$$

$$+a_{37}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\mathcal{W}_1\,\mathcal{W}_2\,\Big\{\gamma^\beta\circ\gamma_\beta\Big\}\,\Big\rangle\!\!\Big\rangle_1+a_{38}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\mathcal{W}_1\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\gamma_\beta\,\mathcal{W}_2\,\gamma^\mu\,\gamma_\mu\,\rangle\!\rangle_0$$

$$+a_{39}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\mathcal{W}_2\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\gamma_\beta\,\mathcal{W}_1\,\gamma^\mu\,\gamma_\mu\,\rangle\!\rangle_0+a_{40}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\mu\,\gamma_\mu\,\rangle\!\rangle_0$$

$$+a_{41}\,\langle\!\langle\,\mathcal{W}_1\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\gamma^\beta\,\{\gamma_\beta\circ\mathcal{W}_2\}\,\Big\rangle\!\!\Big\rangle_1+a_{42}\,\langle\!\langle\,\mathcal{W}_2\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\gamma^\beta\,\{\gamma_\beta\circ\mathcal{W}_1\}\,\Big\rangle\!\!\Big\rangle_1$$

$$+a_{43}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\mathcal{W}_1\,\gamma^\beta\,\{\gamma_\beta\circ\mathcal{W}_2\}\,\Big\rangle\!\!\Big\rangle_1+a_{44}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\mathcal{W}_2\,\gamma^\beta\,\{\gamma_\beta\circ\mathcal{W}_1\}\,\Big\rangle\!\!\Big\rangle_1$$

$$+a_{45}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\gamma^\beta\,\gamma^\mu\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\gamma_\beta\,\gamma_\mu\,\mathcal{W}_1\,\mathcal{W}_2\,\rangle\!\rangle_0+a_{46}\,\Big\langle\!\!\Big\langle\,\mathcal{W}_1\,\gamma^\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\gamma_\alpha\,\{\gamma_\beta\circ\mathcal{W}_2\}\,\rangle\!\rangle_1$$

$$+a_{47}\,\Big\langle\!\!\Big\langle\,\mathcal{W}_2\,\gamma^\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\gamma_\alpha\,\{\gamma_\beta\circ\mathcal{W}_1\}\,\rangle\!\rangle_1+a_{48}\,\Big\langle\!\!\Big\langle\,\gamma^\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\gamma_\alpha\,\gamma^\mu\,\rangle\!\rangle_1\,\langle\!\langle\,\gamma_\mu\,\gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2\,\rangle\!\rangle_0$$

$$+a_{49}\,\Big\langle\!\!\Big\langle\,\gamma^\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\{\mathcal{W}_1\circ\mathcal{W}_2\}\,\rangle\!\rangle_1+a_{50}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\gamma^\beta\,\gamma_\beta\,\{\mathcal{W}_1\circ\mathcal{W}_2\}\,\Big\rangle\!\!\Big\rangle_1$$

$$+a_{51}\,\Big\langle\!\!\Big\langle\,\gamma^\alpha\,\gamma_\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\gamma_\beta\,\{\mathcal{W}_1\circ\mathcal{W}_2\}\,\rangle\!\rangle_1+a_{52}\,\Big\langle\!\!\Big\langle\,\gamma^\alpha\,\gamma_\alpha\,\mathcal{W}_1\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\{\gamma_\beta\circ\mathcal{W}_2\}\,\rangle\!\rangle_1$$

$$+a_{53}\,\Big\langle\!\!\Big\langle\,\gamma^\alpha\,\gamma_\alpha\,\mathcal{W}_2\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\{\gamma_\beta\circ\mathcal{W}_1\}\,\rangle\!\rangle_1+a_{54}\,\Big\langle\!\!\Big\langle\,\gamma^\alpha\,\gamma_\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\mu\,\rangle\!\rangle_0\,\langle\!\langle\,\gamma_\mu\,\rangle\!\rangle_1$$

$$+a_{55}\,\Big\langle\!\!\Big\langle\,\gamma^\alpha\,\gamma_\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\{\gamma_\beta\circ\mathcal{W}_1\}\,\mathcal{W}_2\,\rangle\!\rangle_1+a_{56}\,\Big\langle\!\!\Big\langle\,\gamma^\alpha\,\gamma_\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\{\gamma_\beta\circ\mathcal{W}_2\}\,\mathcal{W}_1\,\rangle\!\rangle_1$$

$$+a_{57}\,\Big\langle\!\!\Big\langle\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\{\gamma_\alpha\circ\gamma_\beta\}\,\rangle\!\rangle_1+a_{58}\,\Big\langle\!\!\Big\langle\,\mathcal{W}_1\,\gamma^\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\mathcal{W}_2\,\gamma^\mu\,\rangle\!\rangle_0\,\langle\!\langle\,\gamma_\mu\,\rangle\!\rangle_1$$

$$+a_{59}\,\Big\langle\!\!\Big\langle\,\mathcal{W}_2\,\gamma^\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\mathcal{W}_1\,\gamma^\mu\,\rangle\!\rangle_0\,\langle\!\langle\,\gamma_\mu\,\rangle\!\rangle_1+a_{60}\,\Big\langle\!\!\Big\langle\,\gamma^\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\mu\,\rangle\!\rangle_0\,\langle\!\langle\,\gamma_\mu\,\rangle\!\rangle_1$$

$$+a_{61}\,\Big\langle\!\!\Big\langle\,\mathcal{W}_1\,\gamma^\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\{\gamma_\alpha\circ\gamma_\beta\}\,\mathcal{W}_2\,\rangle\!\rangle_1+a_{62}\,\Big\langle\!\!\Big\langle\,\mathcal{W}_2\,\gamma^\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\{\gamma_\alpha\circ\gamma_\beta\}\,\mathcal{W}_1\,\rangle\!\rangle_1$$

$$+a_{63}\,\Big\langle\!\!\Big\langle\,\gamma^\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\mathcal{W}_1\,\gamma^\mu\,\rangle\!\rangle_0\,\langle\!\langle\,\gamma_\mu\,\mathcal{W}_2\,\rangle\!\rangle_1+a_{64}\,\Big\langle\!\!\Big\langle\,\gamma^\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\mathcal{W}_2\,\gamma^\mu\,\rangle\!\rangle_0\,\langle\!\langle\,\gamma_\mu\,\mathcal{W}_1\,\rangle\!\rangle_1$$

$$+a_{65}\,\Big\langle\!\!\Big\langle\,\gamma^\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\{\gamma_\alpha\circ\gamma_\beta\}\,\mathcal{W}_1\,\mathcal{W}_2\,\rangle\!\rangle_1+a_{66}\,\langle\!\langle\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\gamma^\beta\,\gamma_\beta\,\gamma^\mu\,\Big\rangle\!\!\Big\rangle_0\,\langle\!\langle\,\gamma_\mu\,\rangle\!\rangle_1$$

$$+a_{67}\,\langle\!\langle\,\mathcal{W}_1\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\mathcal{W}_2\,\gamma^\beta\,\gamma_\beta\,\gamma^\mu\,\Big\rangle\!\!\Big\rangle_0\,\langle\!\langle\,\gamma_\mu\,\rangle\!\rangle_1+a_{68}\,\langle\!\langle\,\mathcal{W}_2\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\mathcal{W}_1\,\gamma^\beta\,\gamma_\beta\,\gamma^\mu\,\Big\rangle\!\!\Big\rangle_0\,\langle\!\langle\,\gamma_\mu\,\rangle\!\rangle_1$$

$$+a_{69}\,\langle\!\langle\,\mathcal{W}_1\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\gamma^\beta\,\gamma_\beta\,\gamma^\mu\,\Big\rangle\!\!\Big\rangle_0\,\langle\!\langle\,\gamma_\mu\,\mathcal{W}_2\,\rangle\!\rangle_1+a_{70}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\beta\,\gamma_\beta\,\gamma^\mu\,\Big\rangle\!\!\Big\rangle_0\,\langle\!\langle\,\gamma_\mu\,\rangle\!\rangle_1$$

$$+a_{71}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\gamma_\beta\,\{\mathcal{W}_1\circ\mathcal{W}_2\}\,\rangle\!\rangle_1+a_{72}\,\langle\!\langle\,\mathcal{W}_1\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\{\gamma_\beta\circ\mathcal{W}_2\}\,\rangle\!\rangle_1$$

$$+a_{73}\,\langle\!\langle\,\mathcal{W}_2\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\{\gamma_\beta\circ\mathcal{W}_1\}\,\rangle\!\rangle_1+a_{74}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\mathcal{W}_1\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\{\gamma_\beta\circ\mathcal{W}_2\}\,\rangle\!\rangle_1$$

$$+a_{75}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\mathcal{W}_2\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\{\gamma_\beta\circ\mathcal{W}_1\}\,\rangle\!\rangle_1+a_{76}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\mu\,\rangle\!\rangle_0\,\langle\!\langle\,\gamma_\mu\,\rangle\!\rangle_1$$

$$+a_{77}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\{\gamma_\beta\circ\mathcal{W}_1\}\,\mathcal{W}_2\,\rangle\!\rangle_1+a_{78}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\,\Big\langle\!\!\Big\langle\,\gamma_\alpha\,\gamma^\beta\,\Big\rangle\!\!\Big\rangle_1\,\langle\!\langle\,\{\gamma_\beta\circ\mathcal{W}_2\}\,\mathcal{W}_1\,\rangle\!\rangle_1$$

$$
\begin{aligned}
&+a_{79} \langle\!\langle \mathcal{W}_1 \mathcal{W}_2 \gamma^\alpha \rangle\!\rangle_1 \left\langle\!\!\left\langle \gamma^\beta \right\rangle\!\!\right\rangle_1 \langle\!\langle \{\gamma_\alpha \circ \gamma_\beta\} \rangle\!\rangle_1 + a_{80} \langle\!\langle \mathcal{W}_1 \gamma^\alpha \rangle\!\rangle_1 \left\langle\!\!\left\langle \gamma^\beta \right\rangle\!\!\right\rangle_1 \langle\!\langle \gamma^\mu \rangle\!\rangle_1 \langle\!\langle \gamma_\alpha \gamma_\beta \gamma_\mu \mathcal{W}_2 \rangle\!\rangle_0 \\
&+a_{81} \langle\!\langle \mathcal{W}_2 \gamma^\alpha \rangle\!\rangle_1 \left\langle\!\!\left\langle \gamma^\beta \right\rangle\!\!\right\rangle_1 \langle\!\langle \gamma^\mu \rangle\!\rangle_1 \langle\!\langle \gamma_\alpha \gamma_\beta \gamma_\mu \mathcal{W}_1 \rangle\!\rangle_0 \\
&+a_{82} \langle\!\langle \gamma^\alpha \rangle\!\rangle_1 \left\langle\!\!\left\langle \gamma^\beta \right\rangle\!\!\right\rangle_1 \langle\!\langle \gamma^\mu \rangle\!\rangle_1 \langle\!\langle \gamma_\alpha \gamma_\beta \gamma_\mu \mathcal{W}_1 \mathcal{W}_2 \rangle\!\rangle_0 \\
&+a_{83} \langle\!\langle \mathcal{W}_1 \gamma^\alpha \rangle\!\rangle_1 \left\langle\!\!\left\langle \mathcal{W}_2 \gamma^\beta \right\rangle\!\!\right\rangle_1 \langle\!\langle \{\gamma_\alpha \circ \gamma_\beta\} \rangle\!\rangle_1 + a_{84} \left\langle\!\!\left\langle \mathcal{W}_1 \mathcal{W}_2 \gamma^\alpha \gamma_\alpha \left\{\gamma^\beta \circ \gamma_\beta\right\} \right\rangle\!\!\right\rangle_1 \\
&+a_{85} \left\langle\!\!\left\langle \mathcal{W}_1 \gamma^\alpha \gamma_\alpha \gamma^\beta \right\rangle\!\!\right\rangle_1 \langle\!\langle \gamma_\beta \mathcal{W}_2 \gamma^\mu \gamma_\mu \rangle\!\rangle_0 + a_{86} \left\langle\!\!\left\langle \mathcal{W}_2 \gamma^\alpha \gamma_\alpha \gamma^\beta \right\rangle\!\!\right\rangle_1 \langle\!\langle \gamma_\beta \mathcal{W}_1 \gamma^\mu \gamma_\mu \rangle\!\rangle_0 \\
&+a_{87} \left\langle\!\!\left\langle \gamma^\alpha \gamma_\alpha \gamma^\beta \right\rangle\!\!\right\rangle_1 \langle\!\langle \gamma_\beta \mathcal{W}_1 \mathcal{W}_2 \gamma^\mu \gamma_\mu \rangle\!\rangle_0 + a_{88} \left\langle\!\!\left\langle \gamma^\alpha \gamma_\alpha \gamma^\beta \gamma_\beta \{\mathcal{W}_1 \circ \mathcal{W}_2\} \right\rangle\!\!\right\rangle_1 \\
&+a_{89} \left\langle\!\!\left\langle \gamma^\alpha \gamma_\alpha \mathcal{W}_1 \gamma^\beta \{\gamma_\beta \circ \mathcal{W}_2\} \right\rangle\!\!\right\rangle_1 + a_{90} \left\langle\!\!\left\langle \gamma^\alpha \gamma_\alpha \mathcal{W}_2 \gamma^\beta \{\gamma_\beta \circ \mathcal{W}_1\} \right\rangle\!\!\right\rangle_1 \\
&+a_{91} \left\langle\!\!\left\langle \gamma^\alpha \gamma_\alpha \gamma^\beta \gamma^\mu \right\rangle\!\!\right\rangle_1 \langle\!\langle \gamma_\beta \gamma_\mu \mathcal{W}_1 \mathcal{W}_2 \rangle\!\rangle_0 + a_{92} \left\langle\!\!\left\langle \mathcal{W}_1 \mathcal{W}_2 \gamma^\alpha \gamma^\beta \{\gamma_\alpha \circ \gamma_\beta\} \right\rangle\!\!\right\rangle_1 \\
&+a_{93} \left\langle\!\!\left\langle \mathcal{W}_1 \gamma^\alpha \gamma^\beta \gamma^\mu \right\rangle\!\!\right\rangle_1 \langle\!\langle \gamma_\alpha \gamma_\beta \gamma_\mu \mathcal{W}_2 \rangle\!\rangle_0 + a_{94} \left\langle\!\!\left\langle \mathcal{W}_2 \gamma^\alpha \gamma^\beta \gamma^\mu \right\rangle\!\!\right\rangle_1 \langle\!\langle \gamma_\alpha \gamma_\beta \gamma_\mu \mathcal{W}_1 \rangle\!\rangle_0 \\
&+a_{95} \left\langle\!\!\left\langle \gamma^\alpha \gamma^\beta \gamma^\mu \right\rangle\!\!\right\rangle_1 \langle\!\langle \gamma_\alpha \gamma_\beta \gamma_\mu \mathcal{W}_1 \mathcal{W}_2 \rangle\!\rangle_0 + a_{96} \left\langle\!\!\left\langle \mathcal{W}_1 \mathcal{W}_2 \gamma^\alpha \gamma^\beta \right\rangle\!\!\right\rangle_1 \langle\!\langle \gamma_\alpha \gamma_\beta \gamma^\mu \gamma_\mu \rangle\!\rangle_0 \\
&+a_{97} \left\langle\!\!\left\langle \mathcal{W}_1 \gamma^\alpha \gamma^\beta \right\rangle\!\!\right\rangle_1 \langle\!\langle \gamma_\alpha \gamma_\beta \mathcal{W}_2 \gamma^\mu \gamma_\mu \rangle\!\rangle_0 + a_{98} \left\langle\!\!\left\langle \mathcal{W}_2 \gamma^\alpha \gamma^\beta \right\rangle\!\!\right\rangle_1 \langle\!\langle \gamma_\alpha \gamma_\beta \mathcal{W}_1 \gamma^\mu \gamma_\mu \rangle\!\rangle_0 \\
&+a_{99} \left\langle\!\!\left\langle \gamma^\alpha \gamma^\beta \right\rangle\!\!\right\rangle_1 \langle\!\langle \gamma_\alpha \gamma_\beta \mathcal{W}_1 \mathcal{W}_2 \gamma^\mu \gamma_\mu \rangle\!\rangle_0 + a_{100} \langle\!\langle \mathcal{W}_1 \mathcal{W}_2 \gamma^\alpha \rangle\!\rangle_1 \left\langle\!\!\left\langle \gamma_\alpha \gamma^\beta \gamma_\beta \gamma^\mu \gamma_\mu \right\rangle\!\!\right\rangle_0 \\
&+a_{101} \langle\!\langle \mathcal{W}_1 \gamma^\alpha \rangle\!\rangle_1 \left\langle\!\!\left\langle \gamma_\alpha \mathcal{W}_2 \gamma^\beta \gamma_\beta \gamma^\mu \gamma_\mu \right\rangle\!\!\right\rangle_0 + a_{102} \langle\!\langle \mathcal{W}_2 \gamma^\alpha \rangle\!\rangle_1 \left\langle\!\!\left\langle \gamma_\alpha \mathcal{W}_1 \gamma^\beta \gamma_\beta \gamma^\mu \gamma_\mu \right\rangle\!\!\right\rangle_0 \\
&+a_{103} \langle\!\langle \gamma^\alpha \rangle\!\rangle_1 \left\langle\!\!\left\langle \gamma_\alpha \mathcal{W}_1 \mathcal{W}_2 \gamma^\beta \gamma_\beta \gamma^\mu \gamma_\mu \right\rangle\!\!\right\rangle_0 + a_{104} \left\langle\!\!\left\langle \mathcal{W}_1 \mathcal{W}_2 \gamma^\alpha \gamma_\alpha \gamma^\beta \gamma_\beta \gamma^\mu \gamma_\mu \right\rangle\!\!\right\rangle_0 \\
&+a_{105} \langle\!\langle \gamma^\alpha \rangle\!\rangle_1 \langle\!\langle \gamma_\alpha \{\mathcal{W}_1 \circ \mathcal{W}_2\} \gamma_\beta \rangle\!\rangle_1 \left\langle\!\!\left\langle \gamma^\beta \right\rangle\!\!\right\rangle_1 \qquad (2)
\end{aligned}
$$

where $\mathcal{W}_i$ are arbitrary vector fields on the big phase space. The following terms are omitted from this formula due to the lower genus equations:

$$
\begin{aligned}
&b_1 \langle\!\langle T(\mathcal{W}_1)\mathcal{W}_2 \{\gamma_\alpha \circ \gamma^\alpha\} \rangle\!\rangle_2 + b_2 \langle\!\langle \mathcal{W}_1 T(\mathcal{W}_2) \{\gamma_\alpha \circ \gamma^\alpha\} \rangle\!\rangle_2 \\
&+b_3 \langle\!\langle T(\mathcal{W}_1) \gamma^\alpha \{\gamma_\alpha \circ \mathcal{W}_2\} \rangle\!\rangle_2 + b_4 \langle\!\langle T(\mathcal{W}_1) \{\mathcal{W}_2 \circ \gamma^\alpha\} \rangle\!\rangle_2 \langle\!\langle \gamma_\alpha \rangle\!\rangle_1 \\
&+b_5 \langle\!\langle \gamma^\alpha \rangle\!\rangle_2 \left\langle\!\!\left\langle \{\gamma_\alpha \circ \mathcal{W}_1\} \mathcal{W}_2 \gamma^\beta \gamma_\beta \right\rangle\!\!\right\rangle_0 + b_6 \langle\!\langle \gamma^\alpha \rangle\!\rangle_2 \left\langle\!\!\left\langle \{\gamma_\alpha \circ \mathcal{W}_2\} \mathcal{W}_1 \gamma^\beta \gamma_\beta \right\rangle\!\!\right\rangle_0 \\
&+b_7 \langle\!\langle \gamma^\alpha \rangle\!\rangle_2 \left\langle\!\!\left\langle \gamma_\alpha \mathcal{W}_2 \gamma^\beta \{\gamma_\beta \circ \mathcal{W}_1\} \right\rangle\!\!\right\rangle_0 . \qquad (3)
\end{aligned}
$$

The b_5, b_6, b_7 terms in this equation are eliminated using the first derivatives of the WDVV equation. To eliminate other terms, we need the Belorousski-Pandharipande equation (abbreviated as BP equation) whose top order terms have the form

$$
\sum_{\sigma \in S_3} \left\langle\!\!\left\langle \mathcal{W}_{\sigma(1)} T(\mathcal{W}_{\sigma(2)} \circ \mathcal{W}_{\sigma(3)}) \right\rangle\!\!\right\rangle_2 - \left\langle\!\!\left\langle T(\mathcal{W}_{\sigma(1)}) \{\mathcal{W}_{\sigma(2)} \circ \mathcal{W}_{\sigma(3)}\} \right\rangle\!\!\right\rangle_2 \;=\; \text{L.O.T.}
$$

for any vector fields $\mathcal{W}_1, \mathcal{W}_2, \mathcal{W}_3$. Throughout this paper, "top order terms" refer to highest order derivatives of the highest genus generating function involved in each equation. "L.O.T." is an abbreviation for "Lower Order Terms". The b_4-term in equation (3) is eliminated by using the BP equation applied to $\mathcal{W}_1$, $\mathcal{W}_2$, γ^α.

Taking covariant derivative of the BP equation with respect to γ^α and applying it to $\mathcal{W}_1, \mathcal{W}_2, \gamma_\alpha$, we obtain

$$\begin{aligned}
&\langle\!\langle\, \gamma^\alpha\, \gamma_\alpha\, T(\mathcal{W}_1 \circ \mathcal{W}_2)\, \rangle\!\rangle_2 - \langle\!\langle\, \gamma^\alpha\, T(\gamma_\alpha)\, \{\mathcal{W}_1 \circ \mathcal{W}_2\}\, \rangle\!\rangle_2 \\
&+ \langle\!\langle\, \gamma^\alpha\, \mathcal{W}_1\, T(\gamma_\alpha \circ \mathcal{W}_2)\, \rangle\!\rangle_2 - \langle\!\langle\, \gamma^\alpha\, T(\mathcal{W}_1)\, \{\gamma_\alpha \circ \mathcal{W}_2\}\, \rangle\!\rangle_2 \\
&+ \langle\!\langle\, \gamma^\alpha\, \mathcal{W}_2\, T(\gamma_\alpha \circ \mathcal{W}_1)\, \rangle\!\rangle_2 - \langle\!\langle\, \gamma^\alpha\, T(\mathcal{W}_2)\, \{\gamma_\alpha \circ \mathcal{W}_1\}\, \rangle\!\rangle_2 = \text{ L.O.T.}
\end{aligned}$$

This equation can be used to eliminate the b_3-term in equation (3).

Taking covariant derivative of the BP equation with respect to $\mathcal{W}_2$ and applying it to $\gamma^\alpha, \gamma_\alpha, \mathcal{W}_1$, we have

$$\langle\!\langle\, \mathcal{W}_1\, \mathcal{W}_2\, T(\gamma^\alpha \circ \gamma_\alpha)\, \rangle\!\rangle_2 - \langle\!\langle\, \mathcal{W}_2\, T(\mathcal{W}_1)\, \{\gamma^\alpha \circ \gamma_\alpha\}\, \rangle\!\rangle_2 \quad = \quad \text{L.O.T.}$$

We can use this equation to eliminate the b_1-term in equation (3). Similarly the b_2-term can be eliminated by using the covariant derivative of the BP equation with respect to $\mathcal{W}_1$.

Since equation (2) holds for Gromov-Witten invariants of all compact symplectic manifolds, it must be satisfied for Gromov-Witten invariants of a point and $\mathbb{P}^1$. It is well known that the Gromov-Witten invariants of these two manifolds satisfy the Virasoro constraints (see [W] [K] for the point case, and [EHX] [Gi] for the $\mathbb{P}^1$ case). A computer program for calculating such invariants based on the Virasoro constraints was written by Gathmann [Ga]. We use Gathmann's program to compute such invariants and plug in derivatives of equation (2) to find relations among constants $a_1, \ldots, a_{105}$.

When the target manifold is a point, the degree of all stable maps must be 0. Hence, we will omit any reference to the degrees of Gromov-Witten invariants of a point. In fact, such Gromov-Witten invariants are just intersection numbers

$$\langle\, \tau_{n_1}\, \cdots\, \tau_{n_k}\, \rangle_g \quad = \quad \int_{\overline{\mathcal{M}}_{g,k}} \psi_1^{n_1} \cdots \psi_k^{n_k}$$

over the moduli spaces $\overline{\mathcal{M}}_{g,k}$. Since the cohomology space of a point is one dimensional, coordinates on the big phase space are simply denoted by $t_0, t_1, t_2, \cdots$. We also identify vector fields $\frac{\partial}{\partial t_m}$ with τ_m on the big phase space. Using Gromov-Witten invariants of a point, we obtained 43 linearly independent relations among constants $a_1, \ldots, a_{105}$. These relations are listed in section A.1 of the appendix as equations (7) to (49).

When the target manifold is $\mathbb{C}P^1$, the degrees of the stable maps are indexed by $H_2(\mathbb{C}P^1;\mathbb{Z}) \cong \mathbb{Z}$. The degree d part of any equation for generating functions of Gromov-Witten invariants is the coefficient of q^d in the Novikov ring. We choose the basis $\{\gamma_0, \gamma_1\}$ for $H^*(\mathbb{C}P^1;\mathbb{C})$ with $\gamma_0 \in H^0(\mathbb{C}P^1;\mathbb{C})$ being the identity of the ordinary cohomology ring and $\gamma_1 \in H^2(\mathbb{C}P^1;\mathbb{C})$ the Poincare dual to a point. Coordinates on the big phase space are denoted by $\{t_n^0, t_n^1 \mid n \in \mathbb{Z}_+\}$. We identify vector fields $\frac{\partial}{\partial t_n^0}$ and $\frac{\partial}{\partial t_n^1}$ with $\tau_{n,0}$ and $\tau_{n,1}$ respectively. Using Gromov-Witten invariants of $\mathbb{P}^1$, we obtained 61 linearly independent relations among constants $a_1, \ldots, a_{105}$. These relations are listed in section A.2 of the appendix as equations (50) to (110).

Combining results from Gromov-Witten invariants of a point and $\mathbb{P}^1$, we obtained 104 linearly independent relations (7) – (110) among $a_1, \ldots, a_{105}$. Using these relations we can solve all a_i with $i \neq 2$ in terms of a_2, and obtain the following formulas:

Lemma 2.1

$$
\begin{array}{llll}
a_1 = 5, & a_2 \text{ is free}, & a_3 = 0, & a_4 = -\frac{1}{36} - 4a_2, \\
a_5 = \frac{5}{168}, & a_6 = \frac{1}{252} - 4a_2, & a_7 = \frac{7}{24} + 6a_2, & a_8 = -\frac{2}{3} - 120a_2, \\
a_9 = \frac{5}{7}, & a_{10} = \frac{2}{21} - 120a_2, & a_{11} = \frac{20}{3} + 240a_2, & a_{12} = \frac{100}{21} + 240a_2, \\
a_{13} = 7 + 120a_2, & a_{14} = -\frac{1}{21} - 6a_2, & a_{15} = \frac{1}{14}, & a_{16} = -\frac{1}{42} - 6a_2, \\
a_{17} = \frac{11}{14} + 24a_2, & a_{18} = \frac{1}{21} + 6a_2, & a_{19} = 0, & a_{20} = -\frac{100}{21} - 240a_2, \\
a_{21} = 0, & a_{22} = \frac{2}{3} + 120a_2, & a_{23} = \frac{1}{36} + 4a_2, & a_{24} = -\frac{1}{252} + 4a_2, \\
a_{25} = -\frac{1}{36} - 5a_2, & a_{26} = -\frac{47}{504} + 4a_2, & a_{27} = 0, & a_{28} = -\frac{7}{12} - 12a_2, \\
a_{29} = -\frac{11}{42} + 6a_2, & a_{30} = 0, & a_{31} = -\frac{47}{21} + 120a_2, & a_{32} = \frac{1}{168} - \frac{3}{5}a_2, \\
a_{33} = -\frac{1}{72} - \frac{8}{5}a_2, & a_{34} = -\frac{11}{504} - \frac{8}{5}a_2, & a_{35} = \frac{97}{2520} + \frac{22}{5}a_2, & \\
a_{36} = \frac{11}{360} + \frac{22}{5}a_2, & a_{37} = -\frac{13}{560} - \frac{11}{10}a_2, & a_{38} = \frac{89}{1680} + \frac{49}{10}a_2, & \\
a_{39} = \frac{97}{1680} + \frac{49}{10}a_2, & a_{40} = -\frac{61}{560} - \frac{71}{10}a_2, & a_{41} = \frac{11}{210} + 6a_2, & \\
a_{42} = \frac{11}{210} + 6a_2, & a_{43} = \frac{8}{105} + 6a_2, & a_{44} = \frac{17}{210} + 6a_2, & a_{45} = -\frac{23}{70} - 24a_2, \\
a_{46} = \frac{12}{35} + 18a_2, & a_{47} = \frac{33}{70} + 18a_2, & a_{48} = -\frac{41}{140} - 36a_2, & a_{49} = -\frac{12}{35} - 18a_2, \\
a_{50} = -\frac{16}{105} - 12a_2, & a_{51} = \frac{19}{210} + 6a_2, & a_{52} = \frac{8}{105} + 6a_2, & a_{53} = \frac{17}{210} + 6a_2, \\
a_{54} = -\frac{3}{35} - 6a_2, & a_{55} = -\frac{17}{105} - 12a_2, & a_{56} = -\frac{1}{7} - 12a_2, & a_{57} = \frac{9}{140} + \frac{36}{5}a_2, \\
a_{58} = -\frac{1}{140} - \frac{54}{5}a_2, & a_{59} = \frac{11}{140} - \frac{54}{5}a_2, & a_{60} = \frac{71}{140} + \frac{36}{5}a_2, & a_{61} = -\frac{13}{140} - \frac{54}{5}a_2, \\
a_{62} = -\frac{7}{60} - \frac{54}{5}a_2, & a_{63} = \frac{9}{28} + \frac{36}{5}a_2, & a_{64} = \frac{71}{420} + \frac{36}{5}a_2, & a_{65} = -\frac{3}{140} + \frac{36}{5}a_2, \\
a_{66} = \frac{59}{1680} + \frac{77}{10}a_2, & a_{67} = \frac{37}{5040} - \frac{33}{10}a_2, & a_{68} = \frac{73}{1008} - \frac{33}{10}a_2, & \\
a_{69} = -\frac{17}{210} - \frac{44}{5}a_2, & a_{70} = \frac{17}{140} + \frac{11}{10}a_2, & a_{71} = \frac{76}{35} + 144a_2, & a_{72} = \frac{44}{35} + 144a_2, \\
a_{73} = \frac{44}{35} + 144a_2, & a_{74} = \frac{64}{35} + 144a_2, & a_{75} = \frac{68}{35} + 144a_2, & a_{76} = -\frac{72}{35} - 144a_2, \\
a_{77} = -\frac{136}{35} - 288a_2, & a_{78} = -\frac{24}{7} - 288a_2, & a_{79} = \frac{22}{35} + 96a_2, & a_{80} = \frac{8}{15} - 24a_2, \\
a_{81} = \frac{148}{105} - 24a_2, & a_{82} = \frac{81}{70} + 12a_2, & a_{83} = -\frac{38}{35} - 168a_2, & a_{84} = -\frac{13}{13440} - \frac{1}{20}a_2, \\
a_{85} = \frac{89}{40320} + \frac{1}{5}a_2, & a_{86} = \frac{97}{40320} + \frac{1}{5}a_2, & a_{87} = -\frac{61}{13440} - \frac{3}{10}a_2, & \\
a_{88} = -\frac{1}{315} - \frac{1}{4}a_2, & a_{89} = \frac{1}{315} + \frac{1}{4}a_2, & a_{90} = \frac{17}{5040} + \frac{1}{4}a_2, & a_{91} = -\frac{23}{1680} - a_2, \\
a_{92} = \frac{1}{630} + \frac{2}{15}a_2, & a_{93} = -\frac{1}{270} - \frac{8}{15}a_2, & a_{94} = -\frac{1}{378} - \frac{8}{15}a_2, & a_{95} = \frac{1}{42} + \frac{4}{5}a_2, \\
a_{96} = \frac{1}{1344} + \frac{3}{10}a_2, & a_{97} = -\frac{1}{448} - \frac{9}{20}a_2, & a_{98} = \frac{3}{2240} - \frac{9}{20}a_2, & \\
a_{99} = \frac{43}{2240} + \frac{3}{10}a_2, & a_{100} = \frac{1}{2688} + \frac{3}{20}a_2, & a_{101} = -\frac{5}{8064} - \frac{1}{10}a_2, & \\
a_{102} = \frac{23}{40320} - \frac{1}{10}a_2, & a_{103} = \frac{11}{2688} + \frac{1}{40}a_2, & a_{104} = \frac{1}{23040}, & a_{105} = -\frac{64}{35} - 144a_2.
\end{array}
$$

After plugging these formulas into equation (2), the coefficient of a_2 is an expression totally symmetric with respect to $\mathcal{W}_1$ and $\mathcal{W}_2$, which can be written as

$$\Omega(\mathcal{W}_1, \mathcal{W}_2) + \Omega(\mathcal{W}_2, \mathcal{W}_1)$$

where

$$
\begin{aligned}
& \Omega(\mathcal{W}_1, \mathcal{W}_2) \\
:= \; & \frac{1}{2} \langle\langle\, \mathcal{W}_1 \, \mathcal{W}_2 \, T(\gamma_\alpha \circ \gamma^\alpha) \,\rangle\rangle_2 - 6 \langle\langle\, \mathcal{W}_1 \, T(\gamma^\alpha \circ \mathcal{W}_2) \, \gamma_\alpha \,\rangle\rangle_2 \\
& + 3 \langle\langle\, T(\gamma^\alpha) \, \gamma_\alpha \, \{\mathcal{W}_1 \circ \mathcal{W}_2\} \,\rangle\rangle_2 + 3 \langle\langle\, \gamma^\alpha \, \{\gamma_\alpha \circ \mathcal{W}_1 \circ \mathcal{W}_2\} \,\rangle\rangle_2 \\
& + 3 \langle\langle\, T(\gamma^\alpha) \,\rangle\rangle_2 \left\langle\!\left\langle\, \gamma_\alpha \, \mathcal{W}_1 \, \mathcal{W}_2 \, \gamma^\beta \, \gamma_\beta \,\right\rangle\!\right\rangle_0 - 4 \langle\langle\, \mathcal{W}_1 \, T(\gamma^\alpha) \,\rangle\rangle_2 \left\langle\!\left\langle\, \gamma_\alpha \, \mathcal{W}_2 \, \gamma^\beta \, \gamma_\beta \,\right\rangle\!\right\rangle_0 \\
& - 120 \langle\langle\, \mathcal{W}_1 \, T(\gamma^\alpha) \,\rangle\rangle_2 \langle\langle\, \{\gamma_\alpha \circ \mathcal{W}_2\} \,\rangle\rangle_1 + 240 \langle\langle\, T(\gamma^\alpha) \,\rangle\rangle_2 \langle\langle\, \{\gamma_\alpha \circ \mathcal{W}_1\} \; \mathcal{W}_2 \,\rangle\rangle_1 \\
& + 60 \langle\langle\, T(\gamma^\alpha) \,\rangle\rangle_2 \left\langle\!\left\langle\, \gamma_\alpha \, \mathcal{W}_1 \, \mathcal{W}_2 \, \gamma^\beta \,\right\rangle\!\right\rangle_0 \langle\langle\, \gamma_\beta \,\rangle\rangle_1 + 12 \left\langle\!\left\langle\, T(\gamma^\alpha) \, \gamma^\beta \,\right\rangle\!\right\rangle_2 \langle\langle\, \gamma_\alpha \, \gamma_\beta \, \mathcal{W}_1 \, \mathcal{W}_2 \,\rangle\rangle_0
\end{aligned}
$$

$$
\begin{aligned}
&-120\,\langle\!\langle\, T(\gamma^\alpha)\,\rangle\!\rangle_2\,\langle\!\langle\,\gamma_\alpha\ \{\mathcal{W}_1\circ\mathcal{W}_2\}\,\rangle\!\rangle_1+60\,\langle\!\langle\,\{\mathcal{W}_1\circ\mathcal{W}_2\}\ T(\gamma^\alpha)\,\rangle\!\rangle_2\,\langle\!\langle\,\gamma_\alpha\,\rangle\!\rangle_1\\
&+4\,\langle\!\langle\,\mathcal{W}_1\ \{\mathcal{W}_2\circ\gamma^\alpha\circ\gamma_\alpha\}\,\rangle\!\rangle_2-\frac{5}{2}\,\langle\!\langle\,\{\mathcal{W}_1\circ\mathcal{W}_2\}\ \{\gamma^\alpha\circ\gamma_\alpha\}\,\rangle\!\rangle_2\\
&+2\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_2\left\langle\!\!\left\langle\,\gamma_\alpha\ \{\mathcal{W}_1\circ\mathcal{W}_2\}\ \gamma^\beta\,\gamma_\beta\,\right\rangle\!\!\right\rangle_0-6\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_2\left\langle\!\!\left\langle\,\left\{\gamma_\alpha\circ\gamma^\beta\right\}\,\gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2\,\right\rangle\!\!\right\rangle_0\\
&+60\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_2\,\langle\!\langle\,\{\gamma_\alpha\circ\mathcal{W}_1\circ\mathcal{W}_2\}\,\rangle\!\rangle_1-\frac{3}{10}\,\langle\!\langle\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\ \left\{\gamma^\beta\circ\gamma_\beta\right\}\,\right\rangle\!\!\right\rangle_1\\
&-\frac{8}{5}\,\langle\!\langle\,\mathcal{W}_1\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\,\mathcal{W}_2\ \left\{\gamma^\beta\circ\gamma_\beta\right\}\,\right\rangle\!\!\right\rangle_1+\frac{22}{5}\,\langle\!\langle\,\mathcal{W}_1\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\beta\,\mathcal{W}_2\,\gamma^\mu\,\gamma_\mu\,\rangle\!\rangle_0\\
&-\frac{11}{20}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\,\mathcal{W}_1\,\mathcal{W}_2\ \left\{\gamma^\beta\circ\gamma_\beta\right\}\,\right\rangle\!\!\right\rangle_1+\frac{49}{10}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\,\mathcal{W}_1\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\beta\,\mathcal{W}_2\,\gamma^\mu\,\gamma_\mu\,\rangle\!\rangle_0\\
&-\frac{71}{20}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\mu\,\gamma_\mu\,\rangle\!\rangle_0+6\,\langle\!\langle\,\mathcal{W}_1\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\,\gamma^\beta\ \{\gamma_\beta\circ\mathcal{W}_2\}\,\right\rangle\!\!\right\rangle_1\\
&+6\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\,\mathcal{W}_1\,\gamma^\beta\ \{\gamma_\beta\circ\mathcal{W}_2\}\,\right\rangle\!\!\right\rangle_1-12\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\,\gamma^\beta\,\gamma^\mu\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\beta\,\gamma_\mu\,\mathcal{W}_1\,\mathcal{W}_2\,\rangle\!\rangle_0\\
&+18\left\langle\!\!\left\langle\,\mathcal{W}_1\,\gamma^\alpha\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\alpha\ \{\gamma_\beta\circ\mathcal{W}_2\}\,\rangle\!\rangle_1-18\left\langle\!\!\left\langle\,\gamma^\alpha\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\alpha\,\gamma^\mu\,\rangle\!\rangle_1\,\langle\!\langle\,\gamma_\mu\,\gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2\,\rangle\!\rangle_0\\
&-9\left\langle\!\!\left\langle\,\gamma^\alpha\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\ \{\mathcal{W}_1\circ\mathcal{W}_2\}\,\rangle\!\rangle_1-6\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\,\gamma^\beta\,\gamma_\beta\ \{\mathcal{W}_1\circ\mathcal{W}_2\}\,\right\rangle\!\!\right\rangle_1\\
&+3\left\langle\!\!\left\langle\,\gamma^\alpha\,\gamma_\alpha\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\beta\ \{\mathcal{W}_1\circ\mathcal{W}_2\}\,\rangle\!\rangle_1+6\left\langle\!\!\left\langle\,\gamma^\alpha\,\gamma_\alpha\,\mathcal{W}_1\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\{\gamma_\beta\circ\mathcal{W}_2\}\,\rangle\!\rangle_1\\
&-3\left\langle\!\!\left\langle\,\gamma^\alpha\,\gamma_\alpha\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\mu\,\rangle\!\rangle_0\,\langle\!\langle\,\gamma_\mu\,\rangle\!\rangle_1-12\left\langle\!\!\left\langle\,\gamma^\alpha\,\gamma_\alpha\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\{\gamma_\beta\circ\mathcal{W}_1\}\ \mathcal{W}_2\,\rangle\!\rangle_1\\
&+\frac{18}{5}\left\langle\!\!\left\langle\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\alpha\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\{\gamma_\alpha\circ\gamma_\beta\}\,\rangle\!\rangle_1-\frac{54}{5}\left\langle\!\!\left\langle\,\mathcal{W}_1\,\gamma^\alpha\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\mathcal{W}_2\,\gamma^\mu\,\rangle\!\rangle_0\,\langle\!\langle\,\gamma_\mu\,\rangle\!\rangle_1\\
&+\frac{18}{5}\left\langle\!\!\left\langle\,\gamma^\alpha\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\mu\,\rangle\!\rangle_0\,\langle\!\langle\,\gamma_\mu\,\rangle\!\rangle_1-\frac{54}{5}\left\langle\!\!\left\langle\,\mathcal{W}_1\,\gamma^\alpha\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\{\gamma_\alpha\circ\gamma_\beta\}\ \mathcal{W}_2\,\rangle\!\rangle_1\\
&+\frac{36}{5}\left\langle\!\!\left\langle\,\gamma^\alpha\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\mathcal{W}_1\,\gamma^\mu\,\rangle\!\rangle_0\,\langle\!\langle\,\gamma_\mu\,\mathcal{W}_2\,\rangle\!\rangle_1+\frac{18}{5}\left\langle\!\!\left\langle\,\gamma^\alpha\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\{\gamma_\alpha\circ\gamma_\beta\}\ \mathcal{W}_1\,\mathcal{W}_2\,\rangle\!\rangle_1\\
&+\frac{77}{20}\,\langle\!\langle\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\,\gamma^\beta\,\gamma_\beta\,\gamma^\mu\,\right\rangle\!\!\right\rangle_0\langle\!\langle\,\gamma_\mu\,\rangle\!\rangle_1-\frac{33}{10}\,\langle\!\langle\,\mathcal{W}_1\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\,\mathcal{W}_2\,\gamma^\beta\,\gamma_\beta\,\gamma^\mu\,\right\rangle\!\!\right\rangle_0\langle\!\langle\,\gamma_\mu\,\rangle\!\rangle_1\\
&-\frac{22}{5}\,\langle\!\langle\,\mathcal{W}_1\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\,\gamma^\beta\,\gamma_\beta\,\gamma^\mu\,\right\rangle\!\!\right\rangle_0\langle\!\langle\,\gamma_\mu\,\mathcal{W}_2\,\rangle\!\rangle_1+\frac{11}{20}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\beta\,\gamma_\beta\,\gamma^\mu\,\right\rangle\!\!\right\rangle_0\langle\!\langle\,\gamma_\mu\,\rangle\!\rangle_1\\
&+72\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\beta\ \{\mathcal{W}_1\circ\mathcal{W}_2\}\,\rangle\!\rangle_1+144\,\langle\!\langle\,\mathcal{W}_1\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\{\gamma_\beta\circ\mathcal{W}_2\}\,\rangle\!\rangle_1\\
&+144\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\,\mathcal{W}_1\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\{\gamma_\beta\circ\mathcal{W}_2\}\,\rangle\!\rangle_1\\
&-72\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\mu\,\rangle\!\rangle_0\,\langle\!\langle\,\gamma_\mu\,\rangle\!\rangle_1\\
&-288\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\{\gamma_\beta\circ\mathcal{W}_1\}\ \mathcal{W}_2\,\rangle\!\rangle_1+48\,\langle\!\langle\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\{\gamma_\alpha\circ\gamma_\beta\}\,\rangle\!\rangle_1\\
&-24\,\langle\!\langle\,\mathcal{W}_1\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma^\mu\,\rangle\!\rangle_1\,\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\gamma_\mu\,\mathcal{W}_2\,\rangle\!\rangle_0\\
&+6\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma^\mu\,\rangle\!\rangle_1\,\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\gamma_\mu\,\mathcal{W}_1\,\mathcal{W}_2\,\rangle\!\rangle_0\\
&-84\,\langle\!\langle\,\mathcal{W}_1\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\mathcal{W}_2\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\{\gamma_\alpha\circ\gamma_\beta\}\,\rangle\!\rangle_1-\frac{1}{40}\left\langle\!\!\left\langle\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\alpha\,\gamma_\alpha\ \left\{\gamma^\beta\circ\gamma_\beta\right\}\,\right\rangle\!\!\right\rangle_1\\
&+\frac{1}{5}\left\langle\!\!\left\langle\,\mathcal{W}_1\,\gamma^\alpha\,\gamma_\alpha\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\beta\,\mathcal{W}_2\,\gamma^\mu\,\gamma_\mu\,\rangle\!\rangle_0-\frac{3}{20}\left\langle\!\!\left\langle\,\gamma^\alpha\,\gamma_\alpha\,\gamma^\beta\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\mu\,\gamma_\mu\,\rangle\!\rangle_0
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{8}\left\langle\!\left\langle\, \gamma^\alpha\,\gamma_\alpha\,\gamma^\beta\,\gamma_\beta\ \{\mathcal{W}_1\circ\mathcal{W}_2\}\,\right\rangle\!\right\rangle_1+\frac{1}{4}\left\langle\!\left\langle\, \gamma^\alpha\,\gamma_\alpha\,\mathcal{W}_1\,\gamma^\beta\ \{\gamma_\beta\circ\mathcal{W}_2\}\,\right\rangle\!\right\rangle_1\\
&-\frac{1}{2}\left\langle\!\left\langle\, \gamma^\alpha\,\gamma_\alpha\,\gamma^\beta\,\gamma^\mu\,\right\rangle\!\right\rangle_1\langle\!\langle\,\gamma_\beta\,\gamma_\mu\,\mathcal{W}_1\,\mathcal{W}_2\,\rangle\!\rangle_0+\frac{1}{15}\left\langle\!\left\langle\, \mathcal{W}_1\,\mathcal{W}_2\,\gamma^\alpha\,\gamma^\beta\ \{\gamma_\alpha\circ\gamma_\beta\}\,\right\rangle\!\right\rangle_1\\
&-\frac{8}{15}\left\langle\!\left\langle\, \mathcal{W}_1\,\gamma^\alpha\,\gamma^\beta\,\gamma^\mu\,\right\rangle\!\right\rangle_1\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\gamma_\mu\,\mathcal{W}_2\,\rangle\!\rangle_0+\frac{2}{5}\left\langle\!\left\langle\, \gamma^\alpha\,\gamma^\beta\,\gamma^\mu\,\right\rangle\!\right\rangle_1\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\gamma_\mu\,\mathcal{W}_1\,\mathcal{W}_2\,\rangle\!\rangle_0\\
&+\frac{3}{20}\left\langle\!\left\langle\, \mathcal{W}_1\,\mathcal{W}_2\,\gamma^\alpha\,\gamma^\beta\,\right\rangle\!\right\rangle_1\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\gamma^\mu\,\gamma_\mu\,\rangle\!\rangle_0-\frac{9}{20}\left\langle\!\left\langle\, \mathcal{W}_1\,\gamma^\alpha\,\gamma^\beta\,\right\rangle\!\right\rangle_1\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\mathcal{W}_2\,\gamma^\mu\,\gamma_\mu\,\rangle\!\rangle_0\\
&+\frac{3}{20}\left\langle\!\left\langle\, \gamma^\alpha\,\gamma^\beta\,\right\rangle\!\right\rangle_1\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\mu\,\gamma_\mu\,\rangle\!\rangle_0+\frac{3}{40}\langle\!\langle\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\left\langle\, \gamma_\alpha\,\gamma^\beta\,\gamma_\beta\,\gamma^\mu\,\gamma_\mu\,\right\rangle\!\right\rangle_0\\
&-\frac{1}{10}\langle\!\langle\,\mathcal{W}_1\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\left\langle\, \gamma_\alpha\,\mathcal{W}_2\,\gamma^\beta\,\gamma_\beta\,\gamma^\mu\,\gamma_\mu\,\right\rangle\!\right\rangle_0+\frac{1}{80}\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\left\langle\, \gamma_\alpha\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\beta\,\gamma_\beta\,\gamma^\mu\,\gamma_\mu\,\right\rangle\!\right\rangle_0\\
&-72\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_1\langle\!\langle\,\gamma_\alpha\ \{\mathcal{W}_1\circ\mathcal{W}_2\}\ \gamma_\beta\,\rangle\!\rangle_1\left\langle\!\left\langle\, \gamma^\beta\,\right\rangle\!\right\rangle_1. \qquad (4)
\end{aligned}
$$

Note that $\Omega(\mathcal{W}_1,\mathcal{W}_2)$ only involves data up to genus-2. Combining the above results, we have proved the following

Theorem 2.2 *For Gromov-Witten invariants of any compact symplectic manifold, we have*

$$
\begin{aligned}
&\langle\!\langle\, T^2(\mathcal{W}_1)T(\mathcal{W}_2)\,\rangle\!\rangle_3\\
=\ &5\,\langle\!\langle\, T^2(\mathcal{W}_1\circ\mathcal{W}_2)\,\rangle\!\rangle_3-\frac{1}{36}\,\langle\!\langle\,\mathcal{W}_1\,T(\gamma^\alpha)\,\rangle\!\rangle_2\left\langle\!\left\langle\, \gamma_\alpha\,\mathcal{W}_2\,\gamma^\beta\,\gamma_\beta\,\right\rangle\!\right\rangle_0\\
&+\frac{5}{168}\,\langle\!\langle\,T(\mathcal{W}_2)\,\gamma^\alpha\,\rangle\!\rangle_2\left\langle\!\left\langle\, \gamma_\alpha\,\mathcal{W}_1\,\gamma^\beta\,\gamma_\beta\,\right\rangle\!\right\rangle_0+\frac{1}{252}\,\langle\!\langle\,\mathcal{W}_2\,T(\gamma^\alpha)\,\rangle\!\rangle_2\left\langle\!\left\langle\, \gamma_\alpha\,\mathcal{W}_1\,\gamma^\beta\,\gamma_\beta\,\right\rangle\!\right\rangle_0\\
&+\frac{7}{24}\,\langle\!\langle\,T(\gamma^\alpha)\,\rangle\!\rangle_2\left\langle\!\left\langle\, \gamma_\alpha\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\beta\,\gamma_\beta\,\right\rangle\!\right\rangle_0-\frac{2}{3}\,\langle\!\langle\,\mathcal{W}_1\,T(\gamma^\alpha)\,\rangle\!\rangle_2\langle\!\langle\,\{\gamma_\alpha\circ\mathcal{W}_2\}\,\rangle\!\rangle_1\\
&+\frac{5}{7}\,\langle\!\langle\,T(\mathcal{W}_2)\,\gamma^\alpha\,\rangle\!\rangle_2\langle\!\langle\,\{\gamma_\alpha\circ\mathcal{W}_1\}\,\rangle\!\rangle_1+\frac{2}{21}\,\langle\!\langle\,\mathcal{W}_2\,T(\gamma^\alpha)\,\rangle\!\rangle_2\langle\!\langle\,\{\gamma_\alpha\circ\mathcal{W}_1\}\,\rangle\!\rangle_1\\
&+\frac{20}{3}\,\langle\!\langle\,T(\gamma^\alpha)\,\rangle\!\rangle_2\langle\!\langle\,\{\gamma_\alpha\circ\mathcal{W}_1\}\ \mathcal{W}_2\,\rangle\!\rangle_1+\frac{100}{21}\,\langle\!\langle\,T(\gamma^\alpha)\,\rangle\!\rangle_2\langle\!\langle\,\{\gamma_\alpha\circ\mathcal{W}_2\}\ \mathcal{W}_1\,\rangle\!\rangle_1\\
&+7\,\langle\!\langle\,T(\gamma^\alpha)\,\rangle\!\rangle_2\left\langle\!\left\langle\, \gamma_\alpha\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\beta\,\right\rangle\!\right\rangle_0\langle\!\langle\,\gamma_\beta\,\rangle\!\rangle_1-\frac{1}{21}\,\langle\!\langle\,\mathcal{W}_1\,T(\gamma^\alpha)\ \{\gamma_\alpha\circ\mathcal{W}_2\}\,\rangle\!\rangle_2\\
&+\frac{1}{14}\,\langle\!\langle\,T(\mathcal{W}_2)\,\gamma^\alpha\ \{\gamma_\alpha\circ\mathcal{W}_1\}\,\rangle\!\rangle_2-\frac{1}{42}\,\langle\!\langle\,\mathcal{W}_2\,T(\gamma^\alpha)\ \{\gamma_\alpha\circ\mathcal{W}_1\}\,\rangle\!\rangle_2\\
&+\frac{11}{14}\,\left\langle\!\left\langle\, T(\gamma^\alpha)\,\gamma^\beta\,\right\rangle\!\right\rangle_2\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2\,\rangle\!\rangle_0+\frac{1}{21}\,\langle\!\langle\,T(\gamma^\alpha)\,\gamma_\alpha\ \{\mathcal{W}_1\circ\mathcal{W}_2\}\,\rangle\!\rangle_2\\
&-\frac{100}{21}\,\langle\!\langle\,T(\gamma^\alpha)\,\rangle\!\rangle_2\langle\!\langle\,\gamma_\alpha\ \{\mathcal{W}_1\circ\mathcal{W}_2\}\,\rangle\!\rangle_1+\frac{2}{3}\,\langle\!\langle\,\{\mathcal{W}_1\circ\mathcal{W}_2\}\ T(\gamma^\alpha)\,\rangle\!\rangle_2\langle\!\langle\,\gamma_\alpha\,\rangle\!\rangle_1\\
&+\frac{1}{36}\,\langle\!\langle\,\mathcal{W}_1\ \{\mathcal{W}_2\circ\gamma^\alpha\circ\gamma_\alpha\}\,\rangle\!\rangle_2-\frac{1}{252}\,\langle\!\langle\,\mathcal{W}_2\ \{\mathcal{W}_1\circ\gamma^\alpha\circ\gamma_\alpha\}\,\rangle\!\rangle_2\\
&-\frac{1}{36}\,\langle\!\langle\,\{\mathcal{W}_1\circ\mathcal{W}_2\}\ \{\gamma^\alpha\circ\gamma_\alpha\}\,\rangle\!\rangle_2-\frac{47}{504}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_2\left\langle\!\left\langle\, \gamma_\alpha\ \{\mathcal{W}_1\circ\mathcal{W}_2\}\ \gamma^\beta\,\gamma_\beta\,\right\rangle\!\right\rangle_0\\
&-\frac{7}{12}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_2\left\langle\!\left\langle\, \left\{\gamma_\alpha\circ\gamma^\beta\right\}\ \gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2\,\right\rangle\!\right\rangle_0-\frac{11}{42}\,\langle\!\langle\,\gamma^\alpha\ \{\gamma_\alpha\circ\mathcal{W}_1\ \circ\mathcal{W}_2\}\,\rangle\!\rangle_2\\
&-\frac{47}{21}\,\langle\!\langle\,\gamma^\alpha\,\rangle\!\rangle_2\langle\!\langle\,\{\gamma_\alpha\circ\mathcal{W}_1\circ\mathcal{W}_2\}\,\rangle\!\rangle_1+\frac{1}{168}\,\langle\!\langle\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\left\langle\, \gamma_\alpha\ \left\{\gamma^\beta\circ\gamma_\beta\right\}\,\right\rangle\!\right\rangle_1\\
&-\frac{1}{72}\,\langle\!\langle\,\mathcal{W}_1\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\left\langle\, \gamma_\alpha\,\mathcal{W}_2\ \left\{\gamma^\beta\circ\gamma_\beta\right\}\,\right\rangle\!\right\rangle_1-\frac{11}{504}\,\langle\!\langle\,\mathcal{W}_2\,\gamma^\alpha\,\rangle\!\rangle_1\left\langle\!\left\langle\, \gamma_\alpha\,\mathcal{W}_1\ \left\{\gamma^\beta\circ\gamma_\beta\right\}\,\right\rangle\!\right\rangle_1
\end{aligned}
$$

$$+\frac{97}{2520}\,\langle\!\langle \mathcal{W}_1\,\gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \gamma_\beta\,\mathcal{W}_2\,\gamma^\mu\,\gamma_\mu \rangle\!\rangle_0 + \frac{11}{360}\,\langle\!\langle \mathcal{W}_2\,\gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \gamma_\beta\,\mathcal{W}_1\,\gamma^\mu\,\gamma_\mu \rangle\!\rangle_0$$

$$-\frac{13}{560}\,\langle\!\langle \gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\mathcal{W}_1\,\mathcal{W}_2\,\Big\{\gamma^\beta\circ\gamma_\beta\Big\} \Big\rangle\!\!\Big\rangle_1 + \frac{89}{1680}\,\langle\!\langle \gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\mathcal{W}_1\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \gamma_\beta\,\mathcal{W}_2\,\gamma^\mu\,\gamma_\mu \rangle\!\rangle_0$$

$$+\frac{97}{1680}\,\langle\!\langle \gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\mathcal{W}_2\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \gamma_\beta\,\mathcal{W}_1\,\gamma^\mu\,\gamma_\mu \rangle\!\rangle_0 - \frac{61}{560}\,\langle\!\langle \gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\mu\,\gamma_\mu \rangle\!\rangle_0$$

$$+\frac{11}{210}\,\langle\!\langle \mathcal{W}_1\,\gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\gamma^\beta\,\{\gamma_\beta\circ\mathcal{W}_2\} \Big\rangle\!\!\Big\rangle_1 + \frac{11}{210}\,\langle\!\langle \mathcal{W}_2\,\gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\gamma^\beta\,\{\gamma_\beta\circ\mathcal{W}_1\} \Big\rangle\!\!\Big\rangle_1$$

$$+\frac{8}{105}\,\langle\!\langle \gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\mathcal{W}_1\,\gamma^\beta\,\{\gamma_\beta\circ\mathcal{W}_2\} \Big\rangle\!\!\Big\rangle_1 + \frac{17}{210}\,\langle\!\langle \gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\mathcal{W}_2\,\gamma^\beta\,\{\gamma_\beta\circ\mathcal{W}_1\} \Big\rangle\!\!\Big\rangle_1$$

$$-\frac{23}{70}\,\langle\!\langle \gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\gamma^\beta\,\gamma^\mu \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \gamma_\beta\,\gamma_\mu\,\mathcal{W}_1\,\mathcal{W}_2 \rangle\!\rangle_0 + \frac{12}{35}\,\Big\langle\!\!\Big\langle \mathcal{W}_1\,\gamma^\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \gamma_\alpha\,\{\gamma_\beta\circ\mathcal{W}_2\} \rangle\!\rangle_1$$

$$+\frac{33}{70}\,\Big\langle\!\!\Big\langle \mathcal{W}_2\,\gamma^\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \gamma_\alpha\,\{\gamma_\beta\circ\mathcal{W}_1\} \rangle\!\rangle_1 - \frac{41}{140}\,\Big\langle\!\!\Big\langle \gamma^\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \gamma_\alpha\,\gamma^\mu \rangle\!\rangle_1 \langle\!\langle \gamma_\mu\,\gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2 \rangle\!\rangle_0$$

$$-\frac{12}{35}\,\Big\langle\!\!\Big\langle \gamma^\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \gamma_\alpha\,\gamma_\beta\,\{\mathcal{W}_1\circ\mathcal{W}_2\} \rangle\!\rangle_1 - \frac{16}{105}\,\langle\!\langle \gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\gamma^\beta\,\gamma_\beta\,\{\mathcal{W}_1\circ\mathcal{W}_2\} \Big\rangle\!\!\Big\rangle_1$$

$$+\frac{19}{210}\,\Big\langle\!\!\Big\langle \gamma^\alpha\,\gamma_\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \gamma_\beta\,\{\mathcal{W}_1\circ\mathcal{W}_2\} \rangle\!\rangle_1 + \frac{8}{105}\,\Big\langle\!\!\Big\langle \gamma^\alpha\,\gamma_\alpha\,\mathcal{W}_1\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \{\gamma_\beta\circ\mathcal{W}_2\} \rangle\!\rangle_1$$

$$+\frac{17}{210}\,\Big\langle\!\!\Big\langle \gamma^\alpha\,\gamma_\alpha\,\mathcal{W}_2\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \{\gamma_\beta\circ\mathcal{W}_1\} \rangle\!\rangle_1 - \frac{3}{35}\,\Big\langle\!\!\Big\langle \gamma^\alpha\,\gamma_\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\mu \rangle\!\rangle_0 \langle\!\langle \gamma_\mu \rangle\!\rangle_1$$

$$-\frac{17}{105}\,\Big\langle\!\!\Big\langle \gamma^\alpha\,\gamma_\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \{\gamma_\beta\circ\mathcal{W}_1\}\,\mathcal{W}_2 \rangle\!\rangle_1 - \frac{1}{7}\,\Big\langle\!\!\Big\langle \gamma^\alpha\,\gamma_\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \{\gamma_\beta\circ\mathcal{W}_2\}\,\mathcal{W}_1 \rangle\!\rangle_1$$

$$+\frac{9}{140}\,\Big\langle\!\!\Big\langle \mathcal{W}_1\,\mathcal{W}_2\,\gamma^\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \{\gamma_\alpha\circ\gamma_\beta\} \rangle\!\rangle_1 - \frac{1}{140}\,\Big\langle\!\!\Big\langle \mathcal{W}_1\,\gamma^\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \gamma_\alpha\,\gamma_\beta\,\mathcal{W}_2\,\gamma^\mu \rangle\!\rangle_0 \langle\!\langle \gamma_\mu \rangle\!\rangle_1$$

$$+\frac{11}{140}\,\Big\langle\!\!\Big\langle \mathcal{W}_2\,\gamma^\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \gamma_\alpha\,\gamma_\beta\,\mathcal{W}_1\,\gamma^\mu \rangle\!\rangle_0 \langle\!\langle \gamma_\mu \rangle\!\rangle_1 + \frac{71}{140}\,\Big\langle\!\!\Big\langle \gamma^\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \gamma_\alpha\,\gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\mu \rangle\!\rangle_0 \langle\!\langle \gamma_\mu \rangle\!\rangle_1$$

$$-\frac{13}{140}\,\Big\langle\!\!\Big\langle \mathcal{W}_1\,\gamma^\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \{\gamma_\alpha\circ\gamma_\beta\}\,\mathcal{W}_2 \rangle\!\rangle_1 - \frac{7}{60}\,\Big\langle\!\!\Big\langle \mathcal{W}_2\,\gamma^\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \{\gamma_\alpha\circ\gamma_\beta\}\,\mathcal{W}_1 \rangle\!\rangle_1$$

$$+\frac{9}{28}\,\Big\langle\!\!\Big\langle \gamma^\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \gamma_\alpha\,\gamma_\beta\,\mathcal{W}_1\,\gamma^\mu \rangle\!\rangle_0 \langle\!\langle \gamma_\mu\,\mathcal{W}_2 \rangle\!\rangle_1 + \frac{71}{420}\,\Big\langle\!\!\Big\langle \gamma^\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \gamma_\alpha\,\gamma_\beta\,\mathcal{W}_2\,\gamma^\mu \rangle\!\rangle_0 \langle\!\langle \gamma_\mu\,\mathcal{W}_1 \rangle\!\rangle_1$$

$$-\frac{3}{140}\,\Big\langle\!\!\Big\langle \gamma^\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \{\gamma_\alpha\circ\gamma_\beta\}\,\mathcal{W}_1\,\mathcal{W}_2 \rangle\!\rangle_1 + \frac{59}{1680}\,\langle\!\langle \mathcal{W}_1\,\mathcal{W}_2\,\gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\gamma^\beta\,\gamma_\beta\,\gamma^\mu \Big\rangle\!\!\Big\rangle_0 \langle\!\langle \gamma_\mu \rangle\!\rangle_1$$

$$+\frac{37}{5040}\,\langle\!\langle \mathcal{W}_1\,\gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\mathcal{W}_2\,\gamma^\beta\,\gamma_\beta\,\gamma^\mu \Big\rangle\!\!\Big\rangle_0 \langle\!\langle \gamma_\mu \rangle\!\rangle_1 + \frac{73}{1008}\,\langle\!\langle \mathcal{W}_2\,\gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\mathcal{W}_1\,\gamma^\beta\,\gamma_\beta\,\gamma^\mu \Big\rangle\!\!\Big\rangle_0 \langle\!\langle \gamma_\mu \rangle\!\rangle_1$$

$$-\frac{17}{210}\,\langle\!\langle \mathcal{W}_1\,\gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\gamma^\beta\,\gamma_\beta\,\gamma^\mu \Big\rangle\!\!\Big\rangle_0 \langle\!\langle \gamma_\mu\,\mathcal{W}_2 \rangle\!\rangle_1 + \frac{17}{140}\,\langle\!\langle \gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\beta\,\gamma_\beta\,\gamma^\mu \Big\rangle\!\!\Big\rangle_0 \langle\!\langle \gamma_\mu \rangle\!\rangle_1$$

$$+\frac{76}{35}\,\langle\!\langle \gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \gamma_\beta\,\{\mathcal{W}_1\circ\mathcal{W}_2\} \rangle\!\rangle_1 + \frac{44}{35}\,\langle\!\langle \mathcal{W}_1\,\gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \{\gamma_\beta\circ\mathcal{W}_2\} \rangle\!\rangle_1$$

$$+\frac{44}{35}\,\langle\!\langle \mathcal{W}_2\,\gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \{\gamma_\beta\circ\mathcal{W}_1\} \rangle\!\rangle_1 + \frac{64}{35}\,\langle\!\langle \gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\mathcal{W}_1\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \{\gamma_\beta\circ\mathcal{W}_2\} \rangle\!\rangle_1$$

$$+\frac{68}{35}\,\langle\!\langle \gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\mathcal{W}_2\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \{\gamma_\beta\circ\mathcal{W}_1\} \rangle\!\rangle_1 - \frac{72}{35}\,\langle\!\langle \gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^\mu \rangle\!\rangle_0 \langle\!\langle \gamma_\mu \rangle\!\rangle_1$$

$$-\frac{136}{35}\,\langle\!\langle \gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \{\gamma_\beta\circ\mathcal{W}_1\}\,\mathcal{W}_2 \rangle\!\rangle_1 - \frac{24}{7}\,\langle\!\langle \gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma_\alpha\,\gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \{\gamma_\beta\circ\mathcal{W}_2\}\,\mathcal{W}_1 \rangle\!\rangle_1$$

$$+\frac{22}{35}\,\langle\!\langle \mathcal{W}_1\,\mathcal{W}_2\,\gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \{\gamma_\alpha\circ\gamma_\beta\} \rangle\!\rangle_1 + \frac{8}{15}\,\langle\!\langle \mathcal{W}_1\,\gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \gamma^\mu \rangle\!\rangle_1 \langle\!\langle \gamma_\alpha\,\gamma_\beta\,\gamma_\mu\,\mathcal{W}_2 \rangle\!\rangle_0$$

$$+\frac{148}{105}\,\langle\!\langle \mathcal{W}_2\,\gamma^\alpha \rangle\!\rangle_1 \Big\langle\!\!\Big\langle \gamma^\beta \Big\rangle\!\!\Big\rangle_1 \langle\!\langle \gamma^\mu \rangle\!\rangle_1 \langle\!\langle \gamma_\alpha\,\gamma_\beta\,\gamma_\mu\,\mathcal{W}_1 \rangle\!\rangle_0$$

$$
\begin{aligned}
&+\frac{81}{70}\,\langle\!\langle\,\gamma^{\alpha}\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma^{\beta}\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma^{\mu}\,\rangle\!\rangle_1\,\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\gamma_\mu\,\mathcal{W}_1\,\mathcal{W}_2\,\rangle\!\rangle_0\\
&-\frac{38}{35}\,\langle\!\langle\,\mathcal{W}_1\,\gamma^{\alpha}\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\mathcal{W}_2\,\gamma^{\beta}\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\{\gamma_\alpha\circ\gamma_\beta\}\,\rangle\!\rangle_1-\frac{13}{13440}\left\langle\!\!\left\langle\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^{\alpha}\,\gamma_\alpha\,\left\{\gamma^{\beta}\circ\gamma_\beta\right\}\,\right\rangle\!\!\right\rangle_1\\
&+\frac{89}{40320}\left\langle\!\!\left\langle\,\mathcal{W}_1\,\gamma^{\alpha}\,\gamma_\alpha\,\gamma^{\beta}\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\beta\,\mathcal{W}_2\,\gamma^{\mu}\,\gamma_\mu\,\rangle\!\rangle_0+\frac{97}{40320}\left\langle\!\!\left\langle\,\mathcal{W}_2\,\gamma^{\alpha}\,\gamma_\alpha\,\gamma^{\beta}\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\beta\,\mathcal{W}_1\,\gamma^{\mu}\,\gamma_\mu\,\rangle\!\rangle_0\\
&-\frac{61}{13440}\left\langle\!\!\left\langle\,\gamma^{\alpha}\,\gamma_\alpha\,\gamma^{\beta}\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^{\mu}\,\gamma_\mu\,\rangle\!\rangle_0-\frac{1}{315}\left\langle\!\!\left\langle\,\gamma^{\alpha}\,\gamma_\alpha\,\gamma^{\beta}\,\gamma_\beta\,\{\mathcal{W}_1\circ\mathcal{W}_2\}\,\right\rangle\!\!\right\rangle_1\\
&+\frac{1}{315}\left\langle\!\!\left\langle\,\gamma^{\alpha}\,\gamma_\alpha\,\mathcal{W}_1\,\gamma^{\beta}\,\{\gamma_\beta\circ\mathcal{W}_2\}\,\right\rangle\!\!\right\rangle_1+\frac{17}{5040}\left\langle\!\!\left\langle\,\gamma^{\alpha}\,\gamma_\alpha\,\mathcal{W}_2\,\gamma^{\beta}\,\{\gamma_\beta\circ\mathcal{W}_1\}\,\right\rangle\!\!\right\rangle_1\\
&-\frac{23}{1680}\left\langle\!\!\left\langle\,\gamma^{\alpha}\,\gamma_\alpha\,\gamma^{\beta}\,\gamma^{\mu}\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\beta\,\gamma_\mu\,\mathcal{W}_1\,\mathcal{W}_2\,\rangle\!\rangle_0+\frac{1}{630}\left\langle\!\!\left\langle\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^{\alpha}\,\gamma^{\beta}\,\{\gamma_\alpha\circ\gamma_\beta\}\,\right\rangle\!\!\right\rangle_1\\
&-\frac{1}{270}\left\langle\!\!\left\langle\,\mathcal{W}_1\,\gamma^{\alpha}\,\gamma^{\beta}\,\gamma^{\mu}\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\gamma_\mu\,\mathcal{W}_2\,\rangle\!\rangle_0-\frac{1}{378}\left\langle\!\!\left\langle\,\mathcal{W}_2\,\gamma^{\alpha}\,\gamma^{\beta}\,\gamma^{\mu}\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\gamma_\mu\,\mathcal{W}_1\,\rangle\!\rangle_0\\
&+\frac{1}{42}\left\langle\!\!\left\langle\,\gamma^{\alpha}\,\gamma^{\beta}\,\gamma^{\mu}\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\gamma_\mu\,\mathcal{W}_1\,\mathcal{W}_2\,\rangle\!\rangle_0+\frac{1}{1344}\left\langle\!\!\left\langle\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^{\alpha}\,\gamma^{\beta}\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\gamma^{\mu}\,\gamma_\mu\,\rangle\!\rangle_0\\
&-\frac{1}{448}\left\langle\!\!\left\langle\,\mathcal{W}_1\,\gamma^{\alpha}\,\gamma^{\beta}\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\mathcal{W}_2\,\gamma^{\mu}\,\gamma_\mu\,\rangle\!\rangle_0+\frac{3}{2240}\left\langle\!\!\left\langle\,\mathcal{W}_2\,\gamma^{\alpha}\,\gamma^{\beta}\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\mathcal{W}_1\,\gamma^{\mu}\,\gamma_\mu\,\rangle\!\rangle_0\\
&+\frac{43}{2240}\left\langle\!\!\left\langle\,\gamma^{\alpha}\,\gamma^{\beta}\,\right\rangle\!\!\right\rangle_1\langle\!\langle\,\gamma_\alpha\,\gamma_\beta\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^{\mu}\,\gamma_\mu\,\rangle\!\rangle_0+\frac{1}{2688}\,\langle\!\langle\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^{\alpha}\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\,\gamma^{\beta}\,\gamma_\beta\,\gamma^{\mu}\,\gamma_\mu\,\right\rangle\!\!\right\rangle_0\\
&-\frac{5}{8064}\,\langle\!\langle\,\mathcal{W}_1\,\gamma^{\alpha}\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\,\mathcal{W}_2\,\gamma^{\beta}\,\gamma_\beta\,\gamma^{\mu}\,\gamma_\mu\,\right\rangle\!\!\right\rangle_0+\frac{23}{40320}\,\langle\!\langle\,\mathcal{W}_2\,\gamma^{\alpha}\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\,\mathcal{W}_1\,\gamma^{\beta}\,\gamma_\beta\,\gamma^{\mu}\,\gamma_\mu\,\right\rangle\!\!\right\rangle_0\\
&+\frac{11}{2688}\,\langle\!\langle\,\gamma^{\alpha}\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma_\alpha\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^{\beta}\,\gamma_\beta\,\gamma^{\mu}\,\gamma_\mu\,\right\rangle\!\!\right\rangle_0+\frac{1}{23040}\left\langle\!\!\left\langle\,\mathcal{W}_1\,\mathcal{W}_2\,\gamma^{\alpha}\,\gamma_\alpha\,\gamma^{\beta}\,\gamma_\beta\,\gamma^{\mu}\,\gamma_\mu\,\right\rangle\!\!\right\rangle_0\\
&-\frac{64}{35}\,\langle\!\langle\,\gamma^{\alpha}\,\rangle\!\rangle_1\,\langle\!\langle\,\gamma_\alpha\,\{\mathcal{W}_1\circ\mathcal{W}_2\}\,\gamma_\beta\,\rangle\!\rangle_1\left\langle\!\!\left\langle\,\gamma^{\beta}\,\right\rangle\!\!\right\rangle_1+a_2\,\{\Omega(\mathcal{W}_1,\mathcal{W}_2)+\Omega(\mathcal{W}_2,\mathcal{W}_1)\}
\end{aligned}
\tag{5}
$$

where $\mathcal{W}_1$ and $\mathcal{W}_2$ are arbitrary vector fields on the big phase space, and a_2 is a constant (see also the remark after Proposition 2.3).

It is surprising that the coefficient of a_2 in the above formula is completely symmetric with respect to $\mathcal{W}_1$ and $\mathcal{W}_2$ but the formula itself is not symmetric (e.g. $a_3 = 0$ but $a_5 \neq 0$). It is also interesting to observe that

$$\Omega(\mathcal{W}_1, T(\mathcal{W}_2)) + \Omega(T(\mathcal{W}_2), \mathcal{W}_1)$$

is also symmetric with respect to $\mathcal{W}_1$ and $\mathcal{W}_2$. The proof of this fact, which is omitted here since it is quite long, uses all known universal equations of genus ≤ 2. These facts give a hint that $\Omega(\mathcal{W}_1, \mathcal{W}_2)+\Omega(\mathcal{W}_2, \mathcal{W}_1)$ might be identically equal to 0. We have testified this using a computer program involving Gromov-Witten invariants of a point, $\mathbb{P}^1$, and $\mathbb{P}^2$. In fact, we can prove this under the assumption that the intersection pairing on $R^3(\overline{\mathcal{M}}_{3,2}) \times R^5(\overline{\mathcal{M}}_{3,2})$ is non-degenerate. We note that this assumption follows from a well known conjecture that the tautological ring $R^*(\overline{\mathcal{M}}_{g,n})$ should be Gorenstein (cf. [FP1] and [P]). It is also remarked in [FP1, p4] that $R^*(\overline{\mathcal{M}}_g)$ is indeed Gorenstein for $g \leq 3$. But currently it is not known whether $R^*(\overline{\mathcal{M}}_{3,2})$ is Gorenstein.

Proposition 2.3 *If the intersection pairing on $R^3(\overline{\mathcal{M}}_{3,2}) \times R^5(\overline{\mathcal{M}}_{3,2})$ is non-degenerate, then*

$$\Omega(\mathcal{W}_1, \mathcal{W}_2) + \Omega(\mathcal{W}_2, \mathcal{W}_1) = 0 \tag{6}$$

for all vector fields $\mathcal{W}_1$ and $\mathcal{W}_2$.

Proof: In [Y], S. Yang calculated the rank of the intersection pairing on $R^3(\overline{\mathcal{M}}_{3,2}) \times R^5(\overline{\mathcal{M}}_{3,2})$ to be 104. Each term in equation (2) corresponds to an element in $R^3(\overline{\mathcal{M}}_{3,2})$. So terms in equation (2) with coefficients $a_1, \ldots, a_{105}$ give 105 elements in $R^3(\overline{\mathcal{M}}_{3,2})$. If the intersection pairing on $R^3(\overline{\mathcal{M}}_{3,2}) \times R^5(\overline{\mathcal{M}}_{3,2})$ is non-degenerate, there must exist a linear relation among these 105 terms. This linear relation gives a universal equation for Gromov-Witten invariants. We can add freely any scalar multiplication of this equation to equation (2). By Theorem 2.2, the only freedom in equation (2) is

$$a_2 \left\{ \Omega(\mathcal{W}_1, \mathcal{W}_2) + \Omega(\mathcal{W}_2, \mathcal{W}_1) \right\}.$$

Therefore we must have $\Omega(\mathcal{W}_1, \mathcal{W}_2) + \Omega(\mathcal{W}_2, \mathcal{W}_1) = 0$. $\Box$

Remark: By Proposition 2.3, we can set $a_2 = 0$ in Theorem 2.2 if it is true that the intersection pairing on $R^3(\overline{\mathcal{M}}_{3,2}) \times R^5(\overline{\mathcal{M}}_{3,2})$ is non-degenerate.

Proof of Theorem 0.1: We can use Theorem 2.2 to compute $\left\langle\!\left\langle\, T^2(\mathcal{W}_1) T(\mathcal{W}_2) \,\right\rangle\!\right\rangle_3$ and $\left\langle\!\left\langle\, T^2(\mathcal{W}_2) T(\mathcal{W}_1) \,\right\rangle\!\right\rangle_3$ separately. The difference of these two terms then proves Theorem 0.1. Note that all symmetric terms in Theorem 2.2 are canceled out in

$$\left\langle\!\left\langle\, T^2(\mathcal{W}_1) T(\mathcal{W}_2) \,\right\rangle\!\right\rangle_3 - \left\langle\!\left\langle\, T^2(\mathcal{W}_2) T(\mathcal{W}_1) \,\right\rangle\!\right\rangle_3.$$

This makes Theorem 0.1 a much simpler formula than Theorem 2.2. We would like to point out that the proof of Theorem 0.1 does not rely on Proposition 2.3. The reason is that

$$a_2 \left\{ \Omega(\mathcal{W}_1, \mathcal{W}_2) + \Omega(\mathcal{W}_2, \mathcal{W}_1) \right\}$$

is symmetric with respect to $\mathcal{W}_1$ and $\mathcal{W}_2$. So this term is canceled in

$$\left\langle\!\left\langle\, T^2(\mathcal{W}_1) T(\mathcal{W}_2) \,\right\rangle\!\right\rangle_3 - \left\langle\!\left\langle\, T^2(\mathcal{W}_2) T(\mathcal{W}_1) \,\right\rangle\!\right\rangle_3$$

without using Proposition 2.3. $\Box$

Remark: It turns out that the genus-3 topological recursion relation proved in [KL] can be derived using Theorem 2.2. To see this, let

$$\mathcal{S} := -\sum_{m,\alpha} \tilde{t}^{\alpha}_{m} \tau_{m-1}(\gamma_\alpha)$$

be the string vector field, where $\tilde{t}^{\alpha}_{m} = t^{\alpha}_{m} - \delta_{\alpha,1}\delta_{m,1}$. Then

$$T(\mathcal{S}) = \mathcal{D} := -\sum_{m,\alpha} \tilde{t}^{\alpha}_{m} \, \tau_m(\gamma_\alpha)$$

is the dilaton vector field (cf. [L1] for a proof of this fact). Applying Theorem 2.2 for $\mathcal{W}_1 = T(\mathcal{W})$ and $\mathcal{W}_2 = \mathcal{S}$, then getting rid of the string and dilaton vector fields by using the string and dilaton equations, we can obtain the main result of [KL]. Note that in this derivation, we also don't need Proposition 2.3 since we can prove

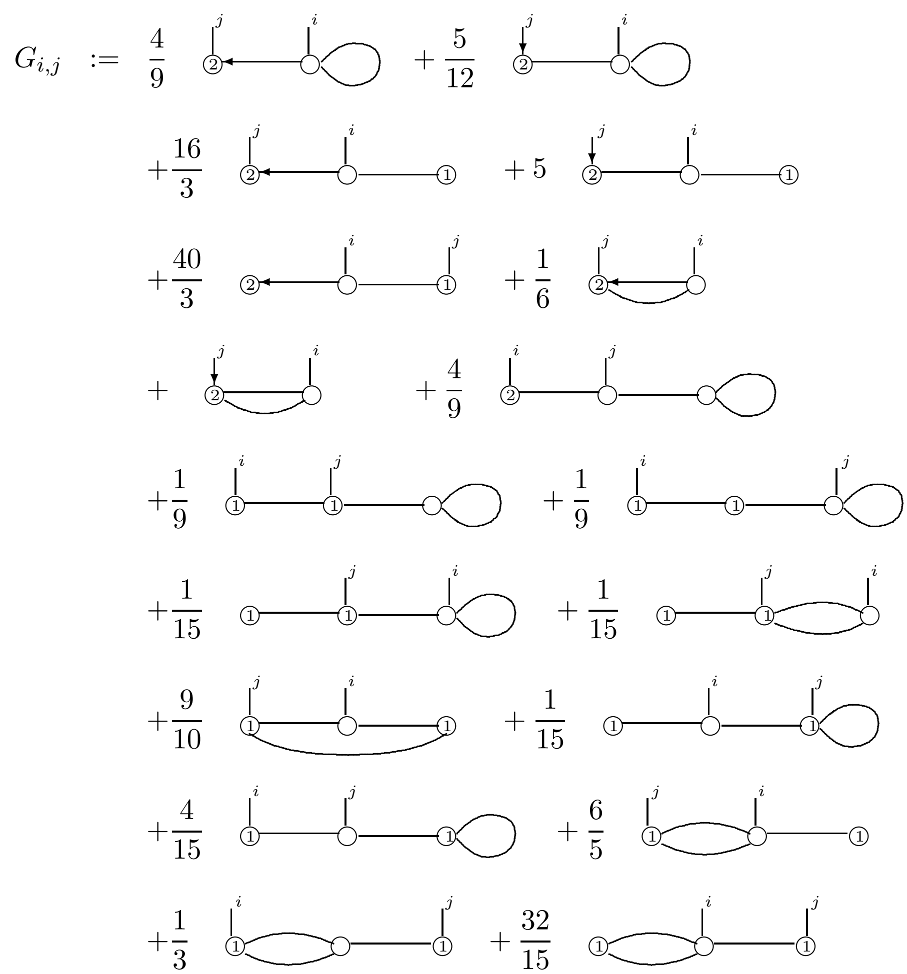
$$\Omega(\mathcal{W}, \mathcal{S}) + \Omega(\mathcal{S}, \mathcal{W}) = \Omega(\mathcal{W}, \mathcal{D}) + \Omega(\mathcal{D}, \mathcal{W}) = 0$$

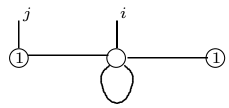
directly using the string and dilaton equations. The proofs of these facts are quite long and are omitted here.

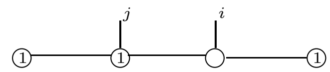
3 New relations in the tautological ring of $\overline{\mathcal{M}}_{3,2}$

Universal equations in Theorems 0.1, Theorem 2.2, and Proposition 2.3 corresponds to 3 relations in the tautological ring of $\overline{\mathcal{M}}_{3,2}$. To describe such relations, we use dual graphs to represent strata of $\overline{\mathcal{M}}_{g,n}$. We adopt the conventions of [Ge2] for dual graphs with a slight modification. We denote vertices of genus 0 by a hollow circle $\bigcirc$, and vertices of genus $g \geq 1$ by ⓖ. A vertex with an incident arrowhead denotes the ψ class associated to the marked point (or a node) on the irreducible component associated to that vertex. When translating relations in the tautological ring of $\overline{\mathcal{M}}_{g,n}$ to universal equations for Gromov-Witten invariants, we need to divide the coefficient of each stratum by the number of elements in the automorphism group of the corresponding dual graph.

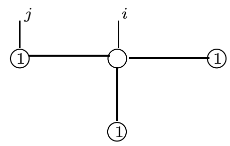
Theorem 2.2 corresponds to a relation representing $\psi_1^2\psi_2$ as a linear combination of 99 boundary classes on $\overline{\mathcal{M}}_{3,2}$. Proposition 2.3 is a linear relation among 69 boundary classes of $\overline{\mathcal{M}}_{3,2}$. Since these two relations are very long, we will omit their dual graph representations in this paper. We only give the dual graph representation for the relation corresponding to Theorem 0.1. The other two relations can be similarly recovered from Proposition 2.3 and Theorem 2.2. Let

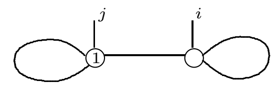


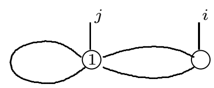
$+\dfrac{41}{45}$ 

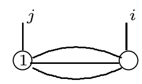
$+\dfrac{4}{5}$ 

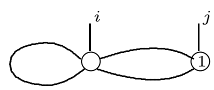
$+\dfrac{16}{5}$ 

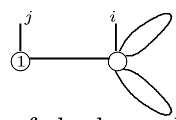
$+\dfrac{184}{15}$ 

$+\dfrac{1}{180}$ 

$+\dfrac{1}{180}$ 

$+\dfrac{2}{45}$ 

$+\dfrac{1}{10}$ 

$+\dfrac{1}{15}$ 

where i and j label the two marked points (i.e. tails of dual graphs). Then Theorem 0.1 corresponds to the following

Theorem 3.1 *In the tautological ring of* $\overline{\mathcal{M}}_{3,2}$, *the following relation holds*

$$\psi_1^2\psi_2 - \psi_1\psi_2^2 \;=\; \frac{1}{7}\left\{G_{1,2} - G_{2,1}\right\}.$$

Appendix

A Relations among constants $a_1, \ldots, a_{105}$

The formulas in Lemma 2.1 are obtained by solving a system of 104 linearly independent relations among constants $a_1, \ldots, a_{105}$ in equation (2). These relations are obtained by using Gromov-Witten theory of a point and $\mathbb{P}^1$. In this appendix, we list all these relations and indicate how they are obtained.

A.1 Relations from the Gromov-Witten invariants of a point

From $\Phi(\tau_0, \tau_5)|_{t=0} = 0$, we obtain

$$\begin{aligned} 0 = {} & \frac{a_2}{288} + \frac{a_{15}}{1152} + \frac{a_{16}}{288} + \frac{a_{24}}{1152} + \frac{a_{84}}{24} + \frac{a_{90}}{24} + \frac{a_{92}}{24} + a_{104} \\ & - \frac{77}{414720}. \end{aligned} \tag{7}$$

From $\Phi(\tau_1, \tau_4)|_{t=0} = 0$, we obtain

$$0 = \frac{a_2}{96} + \frac{a_5}{1152} + \frac{a_6}{384} + \frac{a_{84}}{6} + \frac{a_{86}}{24} + \frac{a_{92}}{6} + \frac{a_{94}}{24} + 5a_{104}$$

$$- \frac{503}{1451520}. \tag{8}$$

From $\Phi(\tau_2, \tau_3)|_{t=0} = 0$, we obtain

$$0 = \frac{29a_2}{1440} + \frac{a_{33}}{576} + \frac{a_{62}}{576} + \frac{7a_{84}}{24} + \frac{7a_{92}}{24} + \frac{a_{98}}{24} + \frac{a_{101}}{24} + 10a_{104} - \frac{607}{1451520}. \tag{9}$$

From $\Phi(\tau_3, \tau_2)|_{t=0} = 0$, we obtain

$$0 = \frac{29a_2}{1440} + \frac{a_{34}}{576} + \frac{a_{61}}{576} + \frac{7a_{84}}{24} + \frac{7a_{92}}{24} + \frac{a_{97}}{24} + \frac{a_{102}}{24} + 10a_{104} - \frac{503}{1451520}. \tag{10}$$

From $\Phi(\tau_4, \tau_1)|_{t=0} = 0$, we obtain

$$0 = \frac{a_2}{96} + \frac{a_3}{1152} + \frac{a_4}{384} + \frac{a_{84}}{6} + \frac{a_{85}}{24} + \frac{a_{92}}{6} + \frac{a_{93}}{24} + 5a_{104} - \frac{77}{414720}. \tag{11}$$

From $\Phi(\tau_5, \tau_0)|_{t=0} = 0$, we obtain

$$0 = \frac{a_2}{288} + \frac{a_{14}}{288} + \frac{a_{23}}{1152} + \frac{a_{84}}{24} + \frac{a_{89}}{24} + \frac{a_{92}}{24} + a_{104} - \frac{5}{82944}. \tag{12}$$

From $\tau_6\Phi(\tau_0, \tau_0)|_{t=0} = 0$, we obtain

$$0 = \frac{77a_1}{414720} + \frac{5a_2}{1152} + \frac{5a_{14}}{1152} + \frac{5a_{15}}{1152} + \frac{5a_{16}}{1152} + \frac{5a_{18}}{1152} + \frac{5a_{19}}{1152} + \frac{a_{23}}{1152} + \frac{a_{24}}{1152} + \frac{a_{25}}{1152} + \frac{a_{29}}{1152} + \frac{a_{84}}{24} + \frac{a_{88}}{24} + \frac{a_{89}}{24} + \frac{a_{90}}{24} + \frac{a_{92}}{24} + a_{104} - \frac{77}{69120}. \tag{13}$$

From $\tau_5\Phi(\tau_0, \tau_1)|_{t=0} = 0$, we obtain

$$0 = \frac{5a_2}{288} + \frac{a_3}{288} + \frac{a_4}{288} + \frac{a_{15}}{90} + \frac{5a_{16}}{288} + \frac{a_{17}}{288} + \frac{a_{24}}{288} + \frac{a_{28}}{1152} + \frac{a_{30}}{1152} + \frac{5a_{84}}{24} + \frac{a_{85}}{24} + \frac{5a_{90}}{24} + \frac{a_{91}}{24} + \frac{5a_{92}}{24} + \frac{a_{93}}{24} + 6a_{104} - \frac{17}{5760}. \tag{14}$$

From $\tau_4\Phi(\tau_0, \tau_2)|_{t=0} = 0$, we obtain

$$0 = \frac{11a_2}{288} + \frac{a_7}{384} + \frac{a_{11}}{9216} + \frac{17a_{15}}{960} + \frac{11a_{16}}{288} + \frac{11a_{24}}{1440} + \frac{a_{34}}{576} + \frac{a_{42}}{576} + \frac{a_{55}}{576} + \frac{a_{61}}{576} + \frac{11a_{84}}{24} + \frac{a_{87}}{24} + \frac{11a_{90}}{24} + \frac{11a_{92}}{24} + \frac{a_{95}}{24} + \frac{a_{97}}{24} + \frac{a_{102}}{24} + 15a_{104} - \frac{1121}{241920}. \tag{15}$$

From $\tau_3\Phi(\tau_0,\tau_3)|_{t=0}=0$, we obtain

$$
\begin{aligned}
0 &= \frac{29a_2}{576}+\frac{17a_{15}}{960}+\frac{29a_{16}}{576}+\frac{29a_{24}}{2880}+\frac{a_{32}}{576}+\frac{a_{33}}{576}+\frac{a_{47}}{576}+\frac{a_{62}}{576}\\
&+\frac{a_{65}}{576}+\frac{7a_{84}}{12}+\frac{7a_{90}}{12}+\frac{7a_{92}}{12}+\frac{a_{98}}{24}+\frac{a_{99}}{24}+\frac{a_{100}}{24}+\frac{a_{101}}{24}\\
&+20a_{104}-\frac{1121}{241920}.
\end{aligned}
\tag{16}
$$

From $\tau_2\Phi(\tau_0,\tau_4)|_{t=0}=0$, we obtain

$$
\begin{aligned}
0 &= \frac{11a_2}{288}+\frac{a_5}{1152}+\frac{a_6}{384}+\frac{a_9}{27648}+\frac{a_{10}}{9216}+\frac{a_{15}}{90}+\frac{11a_{16}}{288}+\frac{11a_{24}}{1440}\\
&+\frac{a_{37}}{576}+\frac{a_{44}}{576}+\frac{a_{53}}{576}+\frac{a_{57}}{576}+\frac{11a_{84}}{24}+\frac{a_{86}}{24}+\frac{11a_{90}}{24}+\frac{11a_{92}}{24}\\
&+\frac{a_{94}}{24}+\frac{a_{96}}{24}+\frac{a_{103}}{24}+15a_{104}-\frac{17}{5760}.
\end{aligned}
\tag{17}
$$

From $\tau_5\Phi(\tau_1,\tau_0)|_{t=0}=0$, we obtain

$$
\begin{aligned}
0 &= \frac{5a_2}{288}+\frac{a_5}{288}+\frac{a_6}{288}+\frac{5a_{14}}{288}+\frac{a_{17}}{288}+\frac{a_{23}}{288}+\frac{a_{27}}{1152}+\frac{a_{28}}{1152}\\
&+\frac{a_{30}}{1152}+\frac{5a_{84}}{24}+\frac{a_{86}}{24}+\frac{5a_{89}}{24}+\frac{a_{91}}{24}+\frac{5a_{92}}{24}+\frac{a_{94}}{24}+6a_{104}\\
&-\frac{503}{241920}.
\end{aligned}
\tag{18}
$$

From $\tau_4\Phi(\tau_1,\tau_1)|_{t=0}=0$, we obtain

$$
\begin{aligned}
0 &= \frac{5a_2}{96}+\frac{11a_3}{1440}+\frac{a_4}{96}+\frac{11a_5}{1440}+\frac{a_6}{96}+\frac{a_7}{192}+\frac{5a_{84}}{6}+\frac{a_{85}}{6}\\
&+\frac{a_{86}}{6}+\frac{a_{87}}{12}+\frac{5a_{92}}{6}+\frac{a_{93}}{6}+\frac{a_{94}}{6}+\frac{a_{95}}{12}+30a_{104}-\frac{1121}{241920}.
\end{aligned}
\tag{19}
$$

From $\tau_3\Phi(\tau_1,\tau_2)|_{t=0}=0$, we obtain

$$
\begin{aligned}
0 &= \frac{29a_2}{288}+\frac{29a_5}{2880}+\frac{29a_6}{1440}+\frac{a_{32}}{288}+\frac{a_{34}}{192}+\frac{a_{36}}{576}+\frac{a_{61}}{192}+\frac{a_{63}}{576}\\
&+\frac{a_{65}}{288}+\frac{35a_{84}}{24}+\frac{7a_{86}}{24}+\frac{35a_{92}}{24}+\frac{7a_{94}}{24}+\frac{a_{97}}{8}+\frac{a_{99}}{8}+\frac{a_{100}}{12}\\
&+\frac{a_{102}}{6}+60a_{104}-\frac{583}{96768}.
\end{aligned}
\tag{20}
$$

From $\tau_2\Phi(\tau_1,\tau_3)|_{t=0}=0$, we obtain

$$
\begin{aligned}
0 &= \frac{29a_2}{288}+\frac{11a_5}{1440}+\frac{29a_6}{1440}+\frac{a_{33}}{288}+\frac{a_{37}}{192}+\frac{a_{39}}{576}+\frac{a_{57}}{192}+\frac{a_{59}}{576}\\
&+\frac{a_{62}}{288}+\frac{35a_{84}}{24}+\frac{7a_{86}}{24}+\frac{35a_{92}}{24}+\frac{7a_{94}}{24}+\frac{a_{96}}{8}+\frac{a_{98}}{8}+\frac{a_{101}}{12}\\
&+\frac{a_{103}}{6}+60a_{104}-\frac{1121}{241920}.
\end{aligned}
\tag{21}
$$

From $\tau_4\Phi(\tau_2,\tau_0)|_{t=0}=0$, we obtain

$$\begin{aligned}0=&\frac{11a_2}{288}+\frac{a_7}{384}+\frac{a_{12}}{9216}+\frac{11a_{14}}{288}+\frac{11a_{23}}{1440}+\frac{a_{33}}{576}+\frac{a_{41}}{576}+\frac{a_{56}}{576}\\&+\frac{a_{62}}{576}+\frac{11a_{84}}{24}+\frac{a_{87}}{24}+\frac{11a_{89}}{24}+\frac{11a_{92}}{24}+\frac{a_{95}}{24}+\frac{a_{98}}{24}+\frac{a_{101}}{24}\\&+15a_{104}-\frac{607}{241920}.\end{aligned}\tag{22}$$

From $\tau_3\Phi(\tau_2,\tau_1)|_{t=0}=0$, we obtain

$$\begin{aligned}0=&\frac{29a_2}{288}+\frac{29a_3}{2880}+\frac{29a_4}{1440}+\frac{a_{32}}{288}+\frac{a_{33}}{192}+\frac{a_{35}}{576}+\frac{a_{62}}{192}+\frac{a_{64}}{576}\\&+\frac{a_{65}}{288}+\frac{35a_{84}}{24}+\frac{7a_{85}}{24}+\frac{35a_{92}}{24}+\frac{7a_{93}}{24}+\frac{a_{98}}{8}+\frac{a_{99}}{8}+\frac{a_{100}}{12}\\&+\frac{a_{101}}{6}+60a_{104}-\frac{1121}{241920}.\end{aligned}\tag{23}$$

From $\tau_2\Phi(\tau_2,\tau_2)|_{t=0}=0$, we obtain

$$\begin{aligned}0=&\frac{7a_2}{48}+\frac{a_{33}}{144}+\frac{a_{34}}{144}+\frac{a_{37}}{144}+\frac{a_{57}}{144}+\frac{a_{61}}{144}+\frac{a_{62}}{144}+\frac{a_{67}}{576}\\&+\frac{a_{68}}{576}+\frac{a_{69}}{576}+\frac{a_{83}}{13824}+2a_{84}+2a_{92}+\frac{a_{96}}{6}+\frac{a_{97}}{6}+\frac{a_{98}}{6}\\&+\frac{a_{101}}{4}+\frac{a_{102}}{4}+\frac{a_{103}}{4}+90a_{104}-\frac{1121}{241920}.\end{aligned}\tag{24}$$

From $\tau_3\Phi(\tau_3,\tau_0)|_{t=0}=0$, we obtain

$$\begin{aligned}0=&\frac{29a_2}{576}+\frac{29a_{14}}{576}+\frac{29a_{23}}{2880}+\frac{a_{32}}{576}+\frac{a_{34}}{576}+\frac{a_{46}}{576}+\frac{a_{61}}{576}+\frac{a_{65}}{576}\\&+\frac{7a_{84}}{12}+\frac{7a_{89}}{12}+\frac{7a_{92}}{12}+\frac{a_{97}}{24}+\frac{a_{99}}{24}+\frac{a_{100}}{24}+\frac{a_{102}}{24}+20a_{104}\\&-\frac{503}{241920}.\end{aligned}\tag{25}$$

From $\tau_2\Phi(\tau_3,\tau_1)|_{t=0}=0$, we obtain

$$\begin{aligned}0=&\frac{29a_2}{288}+\frac{11a_3}{1440}+\frac{29a_4}{1440}+\frac{a_{34}}{288}+\frac{a_{37}}{192}+\frac{a_{38}}{576}+\frac{a_{57}}{192}+\frac{a_{58}}{576}\\&+\frac{a_{61}}{288}+\frac{35a_{84}}{24}+\frac{7a_{85}}{24}+\frac{35a_{92}}{24}+\frac{7a_{93}}{24}+\frac{a_{96}}{8}+\frac{a_{97}}{8}+\frac{a_{102}}{12}\\&+\frac{a_{103}}{6}+60a_{104}-\frac{17}{5760}.\end{aligned}\tag{26}$$

From $\tau_2\Phi(\tau_4,\tau_0)|_{t=0}=0$, we obtain

$$\begin{aligned}0=&\frac{11a_2}{288}+\frac{a_3}{1152}+\frac{a_4}{384}+\frac{a_8}{9216}+\frac{11a_{14}}{288}+\frac{11a_{23}}{1440}+\frac{a_{37}}{576}+\frac{a_{43}}{576}\\&+\frac{a_{52}}{576}+\frac{a_{57}}{576}+\frac{11a_{84}}{24}+\frac{a_{85}}{24}+\frac{11a_{89}}{24}+\frac{11a_{92}}{24}+\frac{a_{93}}{24}+\frac{a_{96}}{24}\\&+\frac{a_{103}}{24}+15a_{104}-\frac{77}{69120}.\end{aligned}\tag{27}$$

From $\tau_3\tau_4\Phi(\tau_0,\tau_0)|_{t=0}=0$, we obtain

$$\begin{aligned}
0 &= \frac{1121a_1}{241920} + \frac{17a_2}{160} + \frac{a_7}{384} + \frac{a_{11}}{9216} + \frac{a_{12}}{9216} + \frac{17a_{14}}{160} + \frac{17a_{15}}{160} + \frac{17a_{16}}{160} \\
&+ \frac{17a_{18}}{160} + \frac{17a_{19}}{160} + \frac{a_{20}}{9216} + \frac{17a_{23}}{960} + \frac{17a_{24}}{960} + \frac{17a_{25}}{960} + \frac{17a_{29}}{960} + \frac{a_{32}}{576} \\
&+ \frac{a_{33}}{576} + \frac{a_{34}}{576} + \frac{a_{41}}{576} + \frac{a_{42}}{576} + \frac{a_{46}}{576} + \frac{a_{47}}{576} + \frac{a_{49}}{576} + \frac{a_{51}}{576} \\
&+ \frac{a_{55}}{576} + \frac{a_{56}}{576} + \frac{a_{61}}{576} + \frac{a_{62}}{576} + \frac{a_{65}}{576} + \frac{25a_{84}}{24} + \frac{a_{87}}{24} + \frac{25a_{88}}{24} \\
&+ \frac{25a_{89}}{24} + \frac{25a_{90}}{24} + \frac{25a_{92}}{24} + \frac{a_{95}}{24} + \frac{a_{97}}{24} + \frac{a_{98}}{24} + \frac{a_{99}}{24} + \frac{a_{100}}{24} \\
&+ \frac{a_{101}}{24} + \frac{a_{102}}{24} + 35a_{104} - \frac{1121}{34560}. \qquad (28)
\end{aligned}$$

From $\tau_2\tau_5\Phi(\tau_0,\tau_0)|_{t=0}=0$, we obtain

$$\begin{aligned}
0 &= \frac{17a_1}{5760} + \frac{a_2}{15} + \frac{a_3}{288} + \frac{a_4}{288} + \frac{a_5}{288} + \frac{a_6}{288} + \frac{a_8}{6912} + \frac{a_9}{6912} \\
&+ \frac{a_{10}}{6912} + \frac{a_{14}}{15} + \frac{a_{15}}{15} + \frac{a_{16}}{15} + \frac{a_{17}}{288} + \frac{a_{18}}{15} + \frac{a_{19}}{15} + \frac{a_{21}}{6912} \\
&+ \frac{a_{22}}{6912} + \frac{a_{23}}{90} + \frac{a_{24}}{90} + \frac{a_{25}}{90} + \frac{a_{26}}{1152} + \frac{a_{27}}{1152} + \frac{a_{28}}{1152} + \frac{a_{29}}{90} \\
&+ \frac{a_{30}}{1152} + \frac{a_{31}}{27648} + \frac{a_{37}}{576} + \frac{a_{43}}{576} + \frac{a_{44}}{576} + \frac{a_{50}}{576} + \frac{a_{52}}{576} + \frac{a_{53}}{576} \\
&+ \frac{a_{57}}{576} + \frac{2a_{84}}{3} + \frac{a_{85}}{24} + \frac{a_{86}}{24} + \frac{2a_{88}}{3} + \frac{2a_{89}}{3} + \frac{2a_{90}}{3} + \frac{a_{91}}{24} \\
&+ \frac{2a_{92}}{3} + \frac{a_{93}}{24} + \frac{a_{94}}{24} + \frac{a_{96}}{24} + \frac{a_{103}}{24} + 21a_{104} - \frac{119}{5760}. \qquad (29)
\end{aligned}$$

From $\tau_3\tau_3\Phi(\tau_0,\tau_1)|_{t=0}=0$, we obtain

$$\begin{aligned}
0 &= \frac{29a_2}{96} + \frac{29a_3}{576} + \frac{29a_4}{576} + \frac{109a_{15}}{576} + \frac{29a_{16}}{96} + \frac{29a_{17}}{576} + \frac{29a_{24}}{576} + \frac{29a_{28}}{2880} \\
&+ \frac{29a_{30}}{2880} + \frac{a_{32}}{96} + \frac{a_{33}}{96} + \frac{a_{35}}{288} + \frac{a_{47}}{96} + \frac{a_{48}}{288} + \frac{a_{62}}{96} + \frac{a_{64}}{288} \\
&+ \frac{a_{65}}{96} + \frac{7a_{84}}{2} + \frac{7a_{85}}{12} + \frac{7a_{90}}{2} + \frac{7a_{91}}{12} + \frac{7a_{92}}{2} + \frac{7a_{93}}{12} + \frac{a_{98}}{4} \\
&+ \frac{a_{99}}{3} + \frac{a_{100}}{4} + \frac{a_{101}}{3} + 140a_{104} - \frac{205}{3456}. \qquad (30)
\end{aligned}$$

From $\tau_2\tau_4\Phi(\tau_0,\tau_1)|_{t=0}=0$, we obtain

$$\begin{aligned}
0 &= \frac{11a_2}{48} + \frac{11a_3}{288} + \frac{11a_4}{288} + \frac{11a_5}{1440} + \frac{a_6}{96} + \frac{a_7}{128} + \frac{11a_9}{34560} + \frac{a_{10}}{2304} \\
&+ \frac{a_{11}}{4608} + \frac{a_{13}}{9216} + \frac{7a_{15}}{48} + \frac{11a_{16}}{48} + \frac{11a_{17}}{288} + \frac{11a_{24}}{288} + \frac{11a_{28}}{1440} + \frac{11a_{30}}{1440} \\
&+ \frac{a_{34}}{288} + \frac{a_{37}}{144} + \frac{a_{38}}{576} + \frac{a_{42}}{288} + \frac{a_{44}}{144} + \frac{a_{45}}{576} + \frac{a_{53}}{144} + \frac{a_{54}}{576} \\
&+ \frac{a_{55}}{288} + \frac{a_{57}}{144} + \frac{a_{58}}{576} + \frac{a_{61}}{288} + \frac{11a_{84}}{4} + \frac{11a_{85}}{24} + \frac{a_{86}}{6} + \frac{a_{87}}{8} \\
&+ \frac{11a_{90}}{4} + \frac{11a_{91}}{24} + \frac{11a_{92}}{4} + \frac{11a_{93}}{24} + \frac{a_{94}}{6} + \frac{a_{95}}{8} + \frac{a_{96}}{6} + \frac{a_{97}}{8}
\end{aligned}$$

$$+\frac{a_{102}}{12}+\frac{5a_{103}}{24}+105a_{104}-\frac{53}{1152}. \tag{31}$$

From $\tau_2\tau_3\Phi(\tau_0,\tau_2)|_{t=0}=0$, we obtain

$$\begin{aligned}0=&\frac{5a_2}{12}+\frac{29a_5}{2880}+\frac{29a_6}{1440}+\frac{29a_7}{1440}+\frac{29a_9}{69120}+\frac{29a_{10}}{34560}+\frac{29a_{11}}{34560}+\frac{109a_{15}}{576}\\&+\frac{5a_{16}}{12}+\frac{5a_{24}}{72}+\frac{a_{32}}{144}+\frac{a_{33}}{144}+\frac{7a_{34}}{576}+\frac{a_{36}}{576}+\frac{7a_{37}}{576}+\frac{a_{40}}{576}\\&+\frac{7a_{42}}{576}+\frac{7a_{44}}{576}+\frac{a_{47}}{144}+\frac{7a_{53}}{576}+\frac{7a_{55}}{576}+\frac{7a_{57}}{576}+\frac{a_{60}}{576}+\frac{7a_{61}}{576}\\&+\frac{a_{62}}{144}+\frac{a_{63}}{576}+\frac{a_{65}}{144}+\frac{a_{67}}{576}+\frac{a_{68}}{576}+\frac{a_{69}}{576}+\frac{a_{73}}{13824}+\frac{a_{77}}{13824}\\&+\frac{a_{83}}{13824}+\frac{59a_{84}}{12}+\frac{7a_{86}}{24}+\frac{7a_{87}}{24}+\frac{59a_{90}}{12}+\frac{59a_{92}}{12}+\frac{7a_{94}}{24}+\frac{7a_{95}}{24}\\&+\frac{7a_{96}}{24}+\frac{7a_{97}}{24}+\frac{a_{98}}{6}+\frac{a_{99}}{4}+\frac{a_{100}}{6}+\frac{a_{101}}{4}+\frac{5a_{102}}{12}+\frac{5a_{103}}{12}\\&+210a_{104}-\frac{205}{3456}.\end{aligned} \tag{32}$$

From $\tau_2\tau_2\Phi(\tau_0,\tau_3)|_{t=0}=0$, we obtain

$$\begin{aligned}0=&\frac{5a_2}{12}+\frac{11a_5}{720}+\frac{29a_6}{720}+\frac{11a_9}{17280}+\frac{29a_{10}}{17280}+\frac{7a_{15}}{48}+\frac{5a_{16}}{12}+\frac{5a_{24}}{72}\\&+\frac{a_{32}}{144}+\frac{a_{33}}{144}+\frac{7a_{37}}{288}+\frac{a_{39}}{288}+\frac{7a_{44}}{288}+\frac{a_{47}}{144}+\frac{7a_{53}}{288}+\frac{7a_{57}}{288}\\&+\frac{a_{59}}{288}+\frac{a_{62}}{144}+\frac{a_{65}}{144}+\frac{a_{66}}{288}+\frac{a_{70}}{288}+\frac{a_{75}}{6912}+\frac{a_{79}}{6912}+\frac{59a_{84}}{12}\\&+\frac{7a_{86}}{12}+\frac{59a_{90}}{12}+\frac{59a_{92}}{12}+\frac{7a_{94}}{12}+\frac{7a_{96}}{12}+\frac{a_{98}}{4}+\frac{a_{99}}{6}+\frac{a_{100}}{4}\\&+\frac{a_{101}}{6}+\frac{5a_{103}}{6}+210a_{104}-\frac{53}{1152}.\end{aligned} \tag{33}$$

From $\tau_3\tau_3\Phi(\tau_1,\tau_0)|_{t=0}=0$, we obtain

$$\begin{aligned}0=&\frac{29a_2}{96}+\frac{29a_5}{576}+\frac{29a_6}{576}+\frac{29a_{14}}{96}+\frac{29a_{17}}{576}+\frac{29a_{23}}{576}+\frac{29a_{27}}{2880}+\frac{29a_{28}}{2880}\\&+\frac{29a_{30}}{2880}+\frac{a_{32}}{96}+\frac{a_{34}}{96}+\frac{a_{36}}{288}+\frac{a_{46}}{96}+\frac{a_{48}}{288}+\frac{a_{61}}{96}+\frac{a_{63}}{288}\\&+\frac{a_{65}}{96}+\frac{7a_{84}}{2}+\frac{7a_{86}}{12}+\frac{7a_{89}}{2}+\frac{7a_{91}}{12}+\frac{7a_{92}}{2}+\frac{7a_{94}}{12}+\frac{a_{97}}{4}\\&+\frac{a_{99}}{3}+\frac{a_{100}}{4}+\frac{a_{102}}{3}+140a_{104}-\frac{583}{13824}.\end{aligned} \tag{34}$$

From $\tau_2\tau_4\Phi(\tau_1,\tau_0)|_{t=0}=0$, we obtain

$$\begin{aligned}0=&\frac{11a_2}{48}+\frac{11a_3}{1440}+\frac{a_4}{96}+\frac{11a_5}{288}+\frac{11a_6}{288}+\frac{a_7}{128}+\frac{a_8}{2304}+\frac{a_{12}}{4608}\\&+\frac{a_{13}}{9216}+\frac{11a_{14}}{48}+\frac{11a_{17}}{288}+\frac{11a_{23}}{288}+\frac{11a_{27}}{1440}+\frac{11a_{28}}{1440}+\frac{11a_{30}}{1440}+\frac{a_{33}}{288}\\&+\frac{a_{37}}{144}+\frac{a_{39}}{576}+\frac{a_{41}}{288}+\frac{a_{43}}{144}+\frac{a_{45}}{576}+\frac{a_{52}}{144}+\frac{a_{54}}{576}+\frac{a_{56}}{288}\end{aligned}$$

$$
\begin{aligned}
&+\frac{a_{57}}{144}+\frac{a_{59}}{576}+\frac{a_{62}}{288}+\frac{11a_{84}}{4}+\frac{a_{85}}{6}+\frac{11a_{86}}{24}+\frac{a_{87}}{8}+\frac{11a_{89}}{4}\\
&+\frac{11a_{91}}{24}+\frac{11a_{92}}{4}+\frac{a_{93}}{6}+\frac{11a_{94}}{24}+\frac{a_{95}}{8}+\frac{a_{96}}{6}+\frac{a_{98}}{8}+\frac{a_{101}}{12}\\
&+\frac{5a_{103}}{24}+105a_{104}-\frac{1121}{34560}. \qquad (35)
\end{aligned}
$$

From $\tau_2\tau_3\Phi(\tau_1,\tau_1)|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{29a_2}{48}+\frac{5a_3}{72}+\frac{29a_4}{288}+\frac{5a_5}{72}+\frac{29a_6}{288}+\frac{29a_7}{720}+\frac{a_{32}}{96}+\frac{a_{33}}{96}\\
&+\frac{a_{34}}{96}+\frac{a_{35}}{288}+\frac{a_{36}}{288}+\frac{a_{37}}{48}+\frac{a_{38}}{192}+\frac{a_{39}}{192}+\frac{a_{40}}{288}+\frac{a_{57}}{48}\\
&+\frac{a_{58}}{192}+\frac{a_{59}}{192}+\frac{a_{60}}{288}+\frac{a_{61}}{96}+\frac{a_{62}}{96}+\frac{a_{63}}{288}+\frac{a_{64}}{288}+\frac{a_{65}}{96}\\
&+\frac{35a_{84}}{4}+\frac{35a_{85}}{24}+\frac{35a_{86}}{24}+\frac{7a_{87}}{12}+\frac{35a_{92}}{4}+\frac{35a_{93}}{24}+\frac{35a_{94}}{24}+\frac{7a_{95}}{12}\\
&+\frac{a_{96}}{2}+\frac{3a_{97}}{8}+\frac{3a_{98}}{8}+\frac{a_{99}}{2}+\frac{a_{100}}{4}+\frac{a_{101}}{3}+\frac{a_{102}}{3}+\frac{5a_{103}}{6}\\
&+420a_{104}-\frac{205}{3456}. \qquad (36)
\end{aligned}
$$

From $\tau_2\tau_2\Phi(\tau_1,\tau_2)|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{7a_2}{8}+\frac{5a_5}{72}+\frac{7a_6}{48}+\frac{a_{32}}{72}+\frac{a_{33}}{36}+\frac{a_{34}}{36}+\frac{a_{36}}{144}+\frac{a_{37}}{18}\\
&+\frac{a_{39}}{72}+\frac{a_{57}}{18}+\frac{a_{59}}{72}+\frac{a_{61}}{36}+\frac{a_{62}}{36}+\frac{a_{63}}{144}+\frac{a_{65}}{72}+\frac{a_{66}}{144}\\
&+\frac{a_{67}}{144}+\frac{a_{68}}{96}+\frac{a_{69}}{144}+\frac{a_{70}}{96}+\frac{a_{79}}{3456}+\frac{a_{81}}{6912}+\frac{a_{83}}{3456}+12a_{84}\\
&+2a_{86}+12a_{92}+2a_{94}+\frac{4a_{96}}{3}+\frac{2a_{97}}{3}+a_{98}+\frac{a_{99}}{2}+\frac{a_{100}}{2}\\
&+a_{101}+\frac{5a_{102}}{4}+\frac{5a_{103}}{2}+630a_{104}-\frac{205}{3456}. \qquad (37)
\end{aligned}
$$

From $\tau_2\tau_3\Phi(\tau_2,\tau_0)|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{5a_2}{12}+\frac{29a_3}{2880}+\frac{29a_4}{1440}+\frac{29a_7}{1440}+\frac{29a_8}{34560}+\frac{29a_{12}}{34560}+\frac{5a_{14}}{12}+\frac{5a_{23}}{72}\\
&+\frac{a_{32}}{144}+\frac{7a_{33}}{576}+\frac{a_{34}}{144}+\frac{a_{35}}{576}+\frac{7a_{37}}{576}+\frac{a_{40}}{576}+\frac{7a_{41}}{576}+\frac{7a_{43}}{576}\\
&+\frac{a_{46}}{144}+\frac{7a_{52}}{576}+\frac{7a_{56}}{576}+\frac{7a_{57}}{576}+\frac{a_{60}}{576}+\frac{a_{61}}{144}+\frac{7a_{62}}{576}+\frac{a_{64}}{576}\\
&+\frac{a_{65}}{144}+\frac{a_{67}}{576}+\frac{a_{68}}{576}+\frac{a_{69}}{576}+\frac{a_{72}}{13824}+\frac{a_{78}}{13824}+\frac{a_{83}}{13824}+\frac{59a_{84}}{12}\\
&+\frac{7a_{85}}{24}+\frac{7a_{87}}{24}+\frac{59a_{89}}{12}+\frac{59a_{92}}{12}+\frac{7a_{93}}{24}+\frac{7a_{95}}{24}+\frac{7a_{96}}{24}+\frac{a_{97}}{6}\\
&+\frac{7a_{98}}{24}+\frac{a_{99}}{4}+\frac{a_{100}}{6}+\frac{5a_{101}}{12}+\frac{a_{102}}{4}+\frac{5a_{103}}{12}+210a_{104}-\frac{1121}{34560}. \qquad (38)
\end{aligned}
$$

From $\tau_2\tau_2\Phi(\tau_2,\tau_1)|_{t=0}=0$, we obtain

$$
0=\frac{7a_2}{8}+\frac{5a_3}{72}+\frac{7a_4}{48}+\frac{a_{32}}{72}+\frac{a_{33}}{36}+\frac{a_{34}}{36}+\frac{a_{35}}{144}+\frac{a_{37}}{18}
$$

$$
\begin{aligned}
&+\frac{a_{38}}{72}+\frac{a_{57}}{18}+\frac{a_{58}}{72}+\frac{a_{61}}{36}+\frac{a_{62}}{36}+\frac{a_{64}}{144}+\frac{a_{65}}{72}+\frac{a_{66}}{144}\\
&+\frac{a_{67}}{96}+\frac{a_{68}}{144}+\frac{a_{69}}{144}+\frac{a_{70}}{96}+\frac{a_{79}}{3456}+\frac{a_{80}}{6912}+\frac{a_{83}}{3456}+12a_{84}\\
&+2a_{85}+12a_{92}+2a_{93}+\frac{4a_{96}}{3}+a_{97}+\frac{2a_{98}}{3}+\frac{a_{99}}{2}+\frac{a_{100}}{2}\\
&+\frac{5a_{101}}{4}+a_{102}+\frac{5a_{103}}{2}+630a_{104}-\frac{53}{1152}.
\end{aligned} \tag{39}
$$

From $\tau_2\tau_2\Phi(\tau_3,\tau_0)|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{5a_2}{12}+\frac{11a_3}{720}+\frac{29a_4}{720}+\frac{29a_8}{17280}+\frac{5a_{14}}{12}+\frac{5a_{23}}{72}+\frac{a_{32}}{144}+\frac{a_{34}}{144}\\
&+\frac{7a_{37}}{288}+\frac{a_{38}}{288}+\frac{7a_{43}}{288}+\frac{a_{46}}{144}+\frac{7a_{52}}{288}+\frac{7a_{57}}{288}+\frac{a_{58}}{288}+\frac{a_{61}}{144}\\
&+\frac{a_{65}}{144}+\frac{a_{66}}{288}+\frac{a_{70}}{288}+\frac{a_{74}}{6912}+\frac{a_{79}}{6912}+\frac{59a_{84}}{12}+\frac{7a_{85}}{12}+\frac{59a_{89}}{12}\\
&+\frac{59a_{92}}{12}+\frac{7a_{93}}{12}+\frac{7a_{96}}{12}+\frac{a_{97}}{4}+\frac{a_{99}}{6}+\frac{a_{100}}{4}+\frac{a_{102}}{6}+\frac{5a_{103}}{6}\\
&+210a_{104}-\frac{119}{5760}.
\end{aligned} \tag{40}
$$

From $\tau_2\tau_3\tau_3\Phi(\tau_0,\tau_0)|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{205a_1}{3456}+\frac{763a_2}{576}+\frac{29a_3}{576}+\frac{29a_4}{576}+\frac{29a_5}{576}+\frac{29a_6}{576}+\frac{29a_7}{720}+\frac{29a_8}{13824}\\
&+\frac{29a_9}{13824}+\frac{29a_{10}}{13824}+\frac{29a_{11}}{17280}+\frac{29a_{12}}{17280}+\frac{763a_{14}}{576}+\frac{763a_{15}}{576}+\frac{763a_{16}}{576}+\frac{29a_{17}}{576}\\
&+\frac{763a_{18}}{576}+\frac{763a_{19}}{576}+\frac{29a_{20}}{17280}+\frac{29a_{21}}{13824}+\frac{29a_{22}}{13824}+\frac{109a_{23}}{576}+\frac{109a_{24}}{576}+\frac{109a_{25}}{576}\\
&+\frac{29a_{26}}{2880}+\frac{29a_{27}}{2880}+\frac{29a_{28}}{2880}+\frac{109a_{29}}{576}+\frac{29a_{30}}{2880}+\frac{29a_{31}}{69120}+\frac{7a_{32}}{288}+\frac{7a_{33}}{288}\\
&+\frac{7a_{34}}{288}+\frac{a_{35}}{288}+\frac{a_{36}}{288}+\frac{7a_{37}}{288}+\frac{a_{40}}{288}+\frac{7a_{41}}{288}+\frac{7a_{42}}{288}+\frac{7a_{43}}{288}\\
&+\frac{7a_{44}}{288}+\frac{7a_{46}}{288}+\frac{7a_{47}}{288}+\frac{a_{48}}{288}+\frac{7a_{49}}{288}+\frac{7a_{50}}{288}+\frac{7a_{51}}{288}+\frac{7a_{52}}{288}\\
&+\frac{7a_{53}}{288}+\frac{7a_{55}}{288}+\frac{7a_{56}}{288}+\frac{7a_{57}}{288}+\frac{a_{60}}{288}+\frac{7a_{61}}{288}+\frac{7a_{62}}{288}+\frac{a_{63}}{288}\\
&+\frac{a_{64}}{288}+\frac{7a_{65}}{288}+\frac{a_{67}}{288}+\frac{a_{68}}{288}+\frac{a_{69}}{288}+\frac{a_{71}}{6912}+\frac{a_{72}}{6912}+\frac{a_{73}}{6912}\\
&+\frac{a_{77}}{6912}+\frac{a_{78}}{6912}+\frac{a_{83}}{6912}+\frac{40a_{84}}{3}+\frac{7a_{85}}{12}+\frac{7a_{86}}{12}+\frac{7a_{87}}{12}+\frac{40a_{88}}{3}\\
&+\frac{40a_{89}}{3}+\frac{40a_{90}}{3}+\frac{7a_{91}}{12}+\frac{40a_{92}}{3}+\frac{7a_{93}}{12}+\frac{7a_{94}}{12}+\frac{7a_{95}}{12}+\frac{7a_{96}}{12}\\
&+\frac{7a_{97}}{12}+\frac{7a_{98}}{12}+\frac{5a_{99}}{6}+\frac{7a_{100}}{12}+\frac{5a_{101}}{6}+\frac{5a_{102}}{6}+\frac{5a_{103}}{6}+560a_{104}\\
&-\frac{205}{432}.
\end{aligned} \tag{41}
$$

From $\tau_2\tau_2\tau_4\Phi(\tau_0,\tau_0)|_{t=0}=0$, we obtain

$$
0=\frac{53a_1}{1152}+\frac{49a_2}{48}+\frac{11a_3}{144}+\frac{11a_4}{144}+\frac{11a_5}{144}+\frac{11a_6}{144}+\frac{a_7}{64}+\frac{11a_8}{3456}
$$

$$
\begin{aligned}
&+\frac{11a_9}{3456}+\frac{11a_{10}}{3456}+\frac{a_{11}}{2304}+\frac{a_{12}}{2304}+\frac{a_{13}}{4608}+\frac{49a_{14}}{48}+\frac{49a_{15}}{48}+\frac{49a_{16}}{48}\\
&+\frac{11a_{17}}{144}+\frac{49a_{18}}{48}+\frac{49a_{19}}{48}+\frac{a_{20}}{2304}+\frac{11a_{21}}{3456}+\frac{11a_{22}}{3456}+\frac{7a_{23}}{48}+\frac{7a_{24}}{48}\\
&+\frac{7a_{25}}{48}+\frac{11a_{26}}{720}+\frac{11a_{27}}{720}+\frac{11a_{28}}{720}+\frac{7a_{29}}{48}+\frac{11a_{30}}{720}+\frac{11a_{31}}{17280}+\frac{a_{32}}{144}\\
&+\frac{a_{33}}{144}+\frac{a_{34}}{144}+\frac{11a_{37}}{288}+\frac{a_{38}}{288}+\frac{a_{39}}{288}+\frac{a_{41}}{144}+\frac{a_{42}}{144}+\frac{11a_{43}}{288}\\
&+\frac{11a_{44}}{288}+\frac{a_{45}}{288}+\frac{a_{46}}{144}+\frac{a_{47}}{144}+\frac{a_{49}}{144}+\frac{11a_{50}}{288}+\frac{a_{51}}{144}+\frac{11a_{52}}{288}\\
&+\frac{11a_{53}}{288}+\frac{a_{54}}{288}+\frac{a_{55}}{144}+\frac{a_{56}}{144}+\frac{11a_{57}}{288}+\frac{a_{58}}{288}+\frac{a_{59}}{288}+\frac{a_{61}}{144}\\
&+\frac{a_{62}}{144}+\frac{a_{65}}{144}+\frac{a_{66}}{288}+\frac{a_{70}}{288}+\frac{a_{74}}{6912}+\frac{a_{75}}{6912}+\frac{a_{79}}{6912}+\frac{125a_{84}}{12}\\
&+\frac{11a_{85}}{12}+\frac{11a_{86}}{12}+\frac{a_{87}}{4}+\frac{125a_{88}}{12}+\frac{125a_{89}}{12}+\frac{125a_{90}}{12}+\frac{11a_{91}}{12}+\frac{125a_{92}}{12}\\
&+\frac{11a_{93}}{12}+\frac{11a_{94}}{12}+\frac{a_{95}}{4}+\frac{11a_{96}}{12}+\frac{a_{97}}{4}+\frac{a_{98}}{4}+\frac{a_{99}}{6}+\frac{a_{100}}{4}\\
&+\frac{a_{101}}{6}+\frac{a_{102}}{6}+\frac{5a_{103}}{4}+420a_{104}+\frac{a_{105}}{6912}-\frac{53}{144}.
\end{aligned}
\tag{42}
$$

From $\tau_2\tau_2\tau_3\Phi(\tau_0,\tau_1)|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{35a_2}{12}+\frac{5a_3}{12}+\frac{5a_4}{12}+\frac{5a_5}{36}+\frac{29a_6}{144}+\frac{29a_7}{240}+\frac{5a_9}{864}+\frac{29a_{10}}{3456}\\
&+\frac{29a_{11}}{8640}+\frac{29a_{13}}{17280}+\frac{11a_{15}}{6}+\frac{35a_{16}}{12}+\frac{5a_{17}}{12}+\frac{5a_{24}}{12}+\frac{5a_{28}}{72}+\frac{5a_{30}}{72}\\
&+\frac{7a_{32}}{144}+\frac{7a_{33}}{144}+\frac{7a_{34}}{144}+\frac{a_{35}}{72}+\frac{a_{36}}{144}+\frac{35a_{37}}{288}+\frac{7a_{38}}{288}+\frac{a_{39}}{96}\\
&+\frac{a_{40}}{96}+\frac{7a_{42}}{144}+\frac{35a_{44}}{288}+\frac{7a_{45}}{288}+\frac{7a_{47}}{144}+\frac{a_{48}}{72}+\frac{35a_{53}}{288}+\frac{7a_{54}}{288}\\
&+\frac{7a_{55}}{144}+\frac{35a_{57}}{288}+\frac{7a_{58}}{288}+\frac{a_{59}}{96}+\frac{a_{60}}{96}+\frac{7a_{61}}{144}+\frac{7a_{62}}{144}+\frac{a_{63}}{144}\\
&+\frac{a_{64}}{72}+\frac{7a_{65}}{144}+\frac{a_{66}}{96}+\frac{a_{67}}{96}+\frac{a_{68}}{144}+\frac{a_{69}}{144}+\frac{a_{70}}{72}+\frac{a_{73}}{3456}\\
&+\frac{a_{75}}{2304}+\frac{a_{76}}{6912}+\frac{a_{77}}{3456}+\frac{a_{79}}{2304}+\frac{a_{80}}{6912}+\frac{a_{83}}{3456}+\frac{413a_{84}}{12}+\frac{59a_{85}}{12}\\
&+\frac{35a_{86}}{12}+\frac{7a_{87}}{4}+\frac{413a_{90}}{12}+\frac{59a_{91}}{12}+\frac{413a_{92}}{12}+\frac{59a_{93}}{12}+\frac{35a_{94}}{12}+\frac{7a_{95}}{4}\\
&+\frac{35a_{96}}{12}+\frac{7a_{97}}{4}+\frac{17a_{98}}{12}+\frac{23a_{99}}{12}+\frac{17a_{100}}{12}+\frac{23a_{101}}{12}+\frac{5a_{102}}{3}+5a_{103}\\
&+1680a_{104}-\frac{193}{288}.
\end{aligned}
\tag{43}
$$

From $\tau_2\tau_2\tau_2\Phi(\tau_0,\tau_2)|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{49a_2}{12}+\frac{5a_5}{24}+\frac{7a_6}{16}+\frac{7a_7}{48}+\frac{5a_9}{576}+\frac{7a_{10}}{384}+\frac{7a_{11}}{1152}+\frac{11a_{15}}{6}\\
&+\frac{49a_{16}}{12}+\frac{7a_{24}}{12}+\frac{a_{32}}{12}+\frac{a_{33}}{12}+\frac{a_{34}}{12}+\frac{a_{36}}{48}+\frac{a_{37}}{4}+\frac{a_{39}}{24}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{a_{40}}{48}+\frac{a_{42}}{12}+\frac{a_{44}}{4}+\frac{a_{47}}{12}+\frac{a_{53}}{4}+\frac{a_{55}}{12}+\frac{a_{57}}{4}+\frac{a_{59}}{24}\\
&+\frac{a_{60}}{48}+\frac{a_{61}}{12}+\frac{a_{62}}{12}+\frac{a_{63}}{48}+\frac{a_{65}}{12}+\frac{a_{66}}{24}+\frac{a_{67}}{48}+\frac{a_{68}}{32}\\
&+\frac{a_{69}}{48}+\frac{a_{70}}{16}+\frac{a_{73}}{1152}+\frac{a_{75}}{576}+\frac{a_{77}}{1152}+\frac{a_{79}}{576}+\frac{a_{81}}{2304}+\frac{a_{82}}{2304}\\
&+\frac{a_{83}}{1152}+48a_{84}+6a_{86}+2a_{87}+48a_{90}+48a_{92}+6a_{94}+2a_{95}\\
&+6a_{96}+2a_{97}+3a_{98}+3a_{99}+3a_{100}+3a_{101}+\frac{15a_{102}}{4}+\frac{45a_{103}}{4}\\
&+2520a_{104}-\frac{193}{288}. \qquad (44)
\end{aligned}
$$

From $\tau_2\tau_2\tau_2\Phi(\tau_1,\tau_1)|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{49a_2}{8}+\frac{7a_3}{12}+\frac{7a_4}{8}+\frac{7a_5}{12}+\frac{7a_6}{8}+\frac{7a_7}{24}+\frac{a_{32}}{8}+\frac{a_{33}}{6}\\
&+\frac{a_{34}}{6}+\frac{a_{35}}{24}+\frac{a_{36}}{24}+\frac{5a_{37}}{12}+\frac{a_{38}}{12}+\frac{a_{39}}{12}+\frac{a_{40}}{24}+\frac{5a_{57}}{12}\\
&+\frac{a_{58}}{12}+\frac{a_{59}}{12}+\frac{a_{60}}{24}+\frac{a_{61}}{6}+\frac{a_{62}}{6}+\frac{a_{63}}{24}+\frac{a_{64}}{24}+\frac{a_{65}}{8}\\
&+\frac{a_{66}}{16}+\frac{a_{67}}{16}+\frac{a_{68}}{16}+\frac{a_{69}}{24}+\frac{a_{70}}{8}+\frac{a_{79}}{384}+\frac{a_{80}}{1152}+\frac{a_{81}}{1152}\\
&+\frac{a_{82}}{1152}+\frac{a_{83}}{576}+84a_{84}+12a_{85}+12a_{86}+4a_{87}+84a_{92}+12a_{93}\\
&+12a_{94}+4a_{95}+10a_{96}+6a_{97}+6a_{98}+6a_{99}+\frac{9a_{100}}{2}+\frac{15a_{101}}{2}\\
&+\frac{15a_{102}}{2}+\frac{45a_{103}}{2}+5040a_{104}-\frac{193}{288}. \qquad (45)
\end{aligned}
$$

From $\tau_2\tau_2\tau_2\Phi(\tau_2,\tau_0)|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{49a_2}{12}+\frac{5a_3}{24}+\frac{7a_4}{16}+\frac{7a_7}{48}+\frac{7a_8}{384}+\frac{7a_{12}}{1152}+\frac{49a_{14}}{12}+\frac{7a_{23}}{12}\\
&+\frac{a_{32}}{12}+\frac{a_{33}}{12}+\frac{a_{34}}{12}+\frac{a_{35}}{48}+\frac{a_{37}}{4}+\frac{a_{38}}{24}+\frac{a_{40}}{48}+\frac{a_{41}}{12}\\
&+\frac{a_{43}}{4}+\frac{a_{46}}{12}+\frac{a_{52}}{4}+\frac{a_{56}}{12}+\frac{a_{57}}{4}+\frac{a_{58}}{24}+\frac{a_{60}}{48}+\frac{a_{61}}{12}\\
&+\frac{a_{62}}{12}+\frac{a_{64}}{48}+\frac{a_{65}}{12}+\frac{a_{66}}{24}+\frac{a_{67}}{32}+\frac{a_{68}}{48}+\frac{a_{69}}{48}+\frac{a_{70}}{16}\\
&+\frac{a_{72}}{1152}+\frac{a_{74}}{576}+\frac{a_{78}}{1152}+\frac{a_{79}}{576}+\frac{a_{80}}{2304}+\frac{a_{82}}{2304}+\frac{a_{83}}{1152}+48a_{84}\\
&+6a_{85}+2a_{87}+48a_{89}+48a_{92}+6a_{93}+2a_{95}+6a_{96}+3a_{97}\\
&+2a_{98}+3a_{99}+3a_{100}+\frac{15a_{101}}{4}+3a_{102}+\frac{45a_{103}}{4}+2520a_{104}-\frac{53}{144}. \qquad (46)
\end{aligned}
$$

From $\tau_2\tau_2\tau_2\tau_3\Phi(\tau_0,\tau_0)|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{193a_1}{288}+\frac{44a_2}{3}+\frac{5a_3}{4}+\frac{5a_4}{4}+\frac{5a_5}{4}+\frac{5a_6}{4}+\frac{61a_7}{120}+\frac{5a_8}{96}\\
&+\frac{5a_9}{96}+\frac{5a_{10}}{96}+\frac{31a_{11}}{1920}+\frac{31a_{12}}{1920}+\frac{29a_{13}}{5760}+\frac{44a_{14}}{3}+\frac{44a_{15}}{3}+\frac{44a_{16}}{3}\\
&+\frac{5a_{17}}{4}+\frac{44a_{18}}{3}+\frac{44a_{19}}{3}+\frac{31a_{20}}{1920}+\frac{5a_{21}}{96}+\frac{5a_{22}}{96}+\frac{11a_{23}}{6}+\frac{11a_{24}}{6}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{11a_{25}}{6}+\frac{5a_{26}}{24}+\frac{5a_{27}}{24}+\frac{5a_{28}}{24}+\frac{11a_{29}}{6}+\frac{5a_{30}}{24}+\frac{5a_{31}}{576}+\frac{11a_{32}}{48}\\
&+\frac{11a_{33}}{48}+\frac{11a_{34}}{48}+\frac{a_{35}}{24}+\frac{a_{36}}{24}+\frac{59a_{37}}{96}+\frac{7a_{38}}{96}+\frac{7a_{39}}{96}+\frac{5a_{40}}{96}\\
&+\frac{11a_{41}}{48}+\frac{11a_{42}}{48}+\frac{59a_{43}}{96}+\frac{59a_{44}}{96}+\frac{7a_{45}}{96}+\frac{11a_{46}}{48}+\frac{11a_{47}}{48}+\frac{a_{48}}{24}\\
&+\frac{11a_{49}}{48}+\frac{59a_{50}}{96}+\frac{11a_{51}}{48}+\frac{59a_{52}}{96}+\frac{59a_{53}}{96}+\frac{7a_{54}}{96}+\frac{11a_{55}}{48}+\frac{11a_{56}}{48}\\
&+\frac{59a_{57}}{96}+\frac{7a_{58}}{96}+\frac{7a_{59}}{96}+\frac{5a_{60}}{96}+\frac{11a_{61}}{48}+\frac{11a_{62}}{48}+\frac{a_{63}}{24}+\frac{a_{64}}{24}\\
&+\frac{11a_{65}}{48}+\frac{7a_{66}}{96}+\frac{5a_{67}}{96}+\frac{5a_{68}}{96}+\frac{a_{69}}{24}+\frac{5a_{70}}{48}+\frac{a_{71}}{576}+\frac{a_{72}}{576}\\
&+\frac{a_{73}}{576}+\frac{7a_{74}}{2304}+\frac{7a_{75}}{2304}+\frac{a_{76}}{2304}+\frac{a_{77}}{576}+\frac{a_{78}}{576}+\frac{7a_{79}}{2304}+\frac{a_{80}}{2304}\\
&+\frac{a_{81}}{2304}+\frac{a_{82}}{2304}+\frac{a_{83}}{576}+\frac{605a_{84}}{4}+\frac{59a_{85}}{4}+\frac{59a_{86}}{4}+\frac{29a_{87}}{4}+\frac{605a_{88}}{4}\\
&+\frac{605a_{89}}{4}+\frac{605a_{90}}{4}+\frac{59a_{91}}{4}+\frac{605a_{92}}{4}+\frac{59a_{93}}{4}+\frac{59a_{94}}{4}+\frac{29a_{95}}{4}+\frac{59a_{96}}{4}\\
&+\frac{29a_{97}}{4}+\frac{29a_{98}}{4}+\frac{35a_{99}}{4}+\frac{29a_{100}}{4}+\frac{35a_{101}}{4}+\frac{35a_{102}}{4}+\frac{105a_{103}}{4}+7560a_{104}\\
&+\frac{7a_{105}}{2304}-\frac{193}{32}.
\end{aligned}
\tag{47}
$$

From $\tau_2\tau_2\tau_2\tau_2\Phi(\tau_0,\tau_1)|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{98a_2}{3}+\frac{49a_3}{12}+\frac{49a_4}{12}+\frac{7a_5}{3}+\frac{7a_6}{2}+\frac{7a_7}{4}+\frac{7a_9}{72}+\frac{7a_{10}}{48}\\
&+\frac{7a_{11}}{144}+\frac{7a_{13}}{288}+\frac{245a_{15}}{12}+\frac{98a_{16}}{3}+\frac{49a_{17}}{12}+\frac{49a_{24}}{12}+\frac{7a_{28}}{12}+\frac{7a_{30}}{12}\\
&+\frac{2a_{32}}{3}+\frac{2a_{33}}{3}+\frac{2a_{34}}{3}+\frac{a_{35}}{6}+\frac{a_{36}}{6}+2a_{37}+\frac{a_{38}}{3}+\frac{a_{39}}{3}\\
&+\frac{a_{40}}{4}+\frac{2a_{42}}{3}+2a_{44}+\frac{a_{45}}{3}+\frac{2a_{47}}{3}+\frac{a_{48}}{6}+2a_{53}+\frac{a_{54}}{3}\\
&+\frac{2a_{55}}{3}+2a_{57}+\frac{a_{58}}{3}+\frac{a_{59}}{3}+\frac{a_{60}}{4}+\frac{2a_{61}}{3}+\frac{2a_{62}}{3}+\frac{a_{63}}{6}\\
&+\frac{a_{64}}{6}+\frac{2a_{65}}{3}+\frac{a_{66}}{3}+\frac{a_{67}}{4}+\frac{a_{68}}{4}+\frac{a_{69}}{6}+\frac{5a_{70}}{8}+\frac{a_{73}}{144}\\
&+\frac{a_{75}}{72}+\frac{a_{76}}{288}+\frac{a_{77}}{144}+\frac{a_{79}}{72}+\frac{a_{80}}{288}+\frac{a_{81}}{288}+\frac{a_{82}}{192}+\frac{a_{83}}{144}\\
&+384a_{84}+48a_{85}+48a_{86}+24a_{87}+384a_{90}+48a_{91}+384a_{92}+48a_{93}\\
&+48a_{94}+24a_{95}+48a_{96}+24a_{97}+24a_{98}+30a_{99}+24a_{100}+30a_{101}\\
&+30a_{102}+105a_{103}+22680a_{104}-\frac{1225}{144}.
\end{aligned}
\tag{48}
$$

From $\tau_2\tau_2\tau_2\tau_2\tau_2\Phi(\tau_0,\tau_0)|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{1225a_1}{144}+\frac{735a_2}{4}+\frac{245a_3}{12}+\frac{245a_4}{12}+\frac{245a_5}{12}+\frac{245a_6}{12}+\frac{35a_7}{4}+\frac{245a_8}{288}\\
&+\frac{245a_9}{288}+\frac{245a_{10}}{288}+\frac{35a_{11}}{144}+\frac{35a_{12}}{144}+\frac{35a_{13}}{288}+\frac{735a_{14}}{4}+\frac{735a_{15}}{4}+\frac{735a_{16}}{4}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{245a_{17}}{12}+\frac{735a_{18}}{4}+\frac{735a_{19}}{4}+\frac{35a_{20}}{144}+\frac{245a_{21}}{288}+\frac{245a_{22}}{288}+\frac{245a_{23}}{12}+\frac{245a_{24}}{12}\\
&+\frac{245a_{25}}{12}+\frac{35a_{26}}{12}+\frac{35a_{27}}{12}+\frac{35a_{28}}{12}+\frac{245a_{29}}{12}+\frac{35a_{30}}{12}+\frac{35a_{31}}{288}+\frac{10a_{32}}{3}\\
&+\frac{10a_{33}}{3}+\frac{10a_{34}}{3}+\frac{5a_{35}}{6}+\frac{5a_{36}}{6}+10a_{37}+\frac{5a_{38}}{3}+\frac{5a_{39}}{3}+\frac{5a_{40}}{4}\\
&+\frac{10a_{41}}{3}+\frac{10a_{42}}{3}+10a_{43}+10a_{44}+\frac{5a_{45}}{3}+\frac{10a_{46}}{3}+\frac{10a_{47}}{3}+\frac{5a_{48}}{6}\\
&+\frac{10a_{49}}{3}+10a_{50}+\frac{10a_{51}}{3}+10a_{52}+10a_{53}+\frac{5a_{54}}{3}+\frac{10a_{55}}{3}+\frac{10a_{56}}{3}\\
&+10a_{57}+\frac{5a_{58}}{3}+\frac{5a_{59}}{3}+\frac{5a_{60}}{4}+\frac{10a_{61}}{3}+\frac{10a_{62}}{3}+\frac{5a_{63}}{6}+\frac{5a_{64}}{6}\\
&+\frac{10a_{65}}{3}+\frac{5a_{66}}{3}+\frac{5a_{67}}{4}+\frac{5a_{68}}{4}+\frac{5a_{69}}{6}+\frac{25a_{70}}{8}+\frac{5a_{71}}{144}+\frac{5a_{72}}{144}\\
&+\frac{5a_{73}}{144}+\frac{5a_{74}}{72}+\frac{5a_{75}}{72}+\frac{5a_{76}}{288}+\frac{5a_{77}}{144}+\frac{5a_{78}}{144}+\frac{5a_{79}}{72}+\frac{5a_{80}}{288}\\
&+\frac{5a_{81}}{288}+\frac{5a_{82}}{192}+\frac{5a_{83}}{144}+1920a_{84}+240a_{85}+240a_{86}+120a_{87}+1920a_{88}\\
&+1920a_{89}+1920a_{90}+240a_{91}+1920a_{92}+240a_{93}+240a_{94}+120a_{95}+240a_{96}\\
&+120a_{97}+120a_{98}+150a_{99}+120a_{100}+150a_{101}+150a_{102}+525a_{103}+113400a_{104}\\
&+\frac{5a_{105}}{72}-\frac{6125}{72}.
\end{aligned}\tag{49}
$$

A.2 Relations from the Gromov-Witten invariants of a $\mathbb{C}P^1$

From the degree 0 part of $\Phi(\tau_{0,0},\tau_{2,1})|_{t=0}=0$, we obtain

$$
0=-\frac{7a_9}{138240}-\frac{7a_{10}}{46080}-\frac{a_{75}}{13824}-\frac{a_{79}}{13824}-\frac{a_{82}}{13824}+\frac{31}{96768}.\tag{50}
$$

From the degree 0 part of $\Phi(\tau_{0,0},\tau_{3,0})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{7a_2}{720}+\frac{a_9}{5760}+\frac{a_{10}}{1920}+\frac{7a_{15}}{2880}+\frac{7a_{16}}{720}+\frac{7a_{24}}{2880}+\frac{a_{37}}{288}+\frac{a_{44}}{288}\\
&+\frac{a_{53}}{288}+\frac{a_{57}}{288}+\frac{a_{70}}{288}+\frac{a_{75}}{6912}+\frac{a_{79}}{6912}-\frac{2329}{1451520}.
\end{aligned}\tag{51}
$$

From the degree 0 part of $\Phi(\tau_{1,0},\tau_{1,1})|_{t=0}=0$, we obtain

$$
0=-\frac{a_{79}}{6912}-\frac{a_{80}}{13824}-\frac{a_{81}}{13824}-\frac{a_{82}}{6912}-\frac{a_{83}}{13824}+\frac{31}{96768}.\tag{52}
$$

From the degree 0 part of $\Phi(\tau_{1,0},\tau_{2,0})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{7a_2}{240}+\frac{7a_5}{2880}+\frac{7a_6}{960}+\frac{a_{33}}{288}+\frac{a_{37}}{96}+\frac{a_{39}}{288}+\frac{a_{57}}{96}+\frac{a_{59}}{288}\\
&+\frac{a_{62}}{288}+\frac{a_{67}}{288}+\frac{a_{70}}{96}+\frac{a_{79}}{3456}+\frac{a_{81}}{6912}+\frac{a_{83}}{6912}-\frac{1501}{725760}.
\end{aligned}\tag{53}
$$

From the degree 0 part of $\Phi(\tau_{1,1},\tau_{1,0})|_{t=0}=0$, we obtain

$$
0=-\frac{a_{79}}{6912}-\frac{a_{80}}{13824}-\frac{a_{81}}{13824}-\frac{a_{82}}{6912}-\frac{a_{83}}{13824}+\frac{31}{96768}.\tag{54}
$$

From the degree 0 part of $\Phi(\tau_{2,0},\tau_{0,1})|_{t=0}=0$, we obtain

$$0=-\frac{7a_8}{46080}-\frac{a_{74}}{13824}-\frac{a_{79}}{13824}-\frac{a_{82}}{13824}+\frac{31}{193536}. \tag{55}$$

From the degree 0 part of $\Phi(\tau_{3,0},\tau_{0,0})|_{t=0}=0$, we obtain

$$\begin{aligned}0=&\frac{7a_2}{720}+\frac{a_8}{1920}+\frac{7a_{14}}{720}+\frac{7a_{23}}{2880}+\frac{a_{37}}{288}+\frac{a_{43}}{288}+\frac{a_{52}}{288}+\frac{a_{57}}{288}\\&+\frac{a_{70}}{288}+\frac{a_{74}}{6912}+\frac{a_{79}}{6912}-\frac{205}{290304}.\end{aligned} \tag{56}$$

From the degree 0 part of $\tau_{3,1}\Phi(\tau_{0,0},\tau_{0,0})|_{t=0}=0$, we obtain

$$\begin{aligned}0=&-\frac{31a_1}{96768}-\frac{7a_8}{34560}-\frac{7a_9}{34560}-\frac{7a_{10}}{34560}-\frac{7a_{21}}{34560}-\frac{7a_{22}}{34560}-\frac{7a_{31}}{138240}\\&-\frac{a_{74}}{13824}-\frac{a_{75}}{13824}-\frac{a_{79}}{13824}-\frac{a_{82}}{13824}-\frac{a_{105}}{13824}+\frac{31}{16128}.\end{aligned} \tag{57}$$

From the degree 0 part of $\tau_{4,0}\Phi(\tau_{0,0},\tau_{0,0})|_{t=0}=0$, we obtain

$$\begin{aligned}0=&\frac{2329a_1}{1451520}+\frac{7a_2}{576}+\frac{a_8}{1440}+\frac{a_9}{1440}+\frac{a_{10}}{1440}+\frac{7a_{14}}{576}+\frac{7a_{15}}{576}+\frac{7a_{16}}{576}\\&+\frac{7a_{18}}{576}+\frac{7a_{19}}{576}+\frac{a_{21}}{1440}+\frac{a_{22}}{1440}+\frac{7a_{23}}{2880}+\frac{7a_{24}}{2880}+\frac{7a_{25}}{2880}+\frac{7a_{29}}{2880}\\&+\frac{a_{31}}{5760}+\frac{a_{37}}{288}+\frac{a_{43}}{288}+\frac{a_{44}}{288}+\frac{a_{50}}{288}+\frac{a_{52}}{288}+\frac{a_{53}}{288}+\frac{a_{57}}{288}\\&+\frac{a_{70}}{288}+\frac{a_{74}}{6912}+\frac{a_{75}}{6912}+\frac{a_{79}}{6912}+\frac{a_{105}}{6912}-\frac{2329}{241920}.\end{aligned} \tag{58}$$

From the degree 0 part of $\tau_{2,1}\Phi(\tau_{0,0},\tau_{1,0})|_{t=0}=0$, we obtain

$$\begin{aligned}0=&-\frac{7a_9}{23040}-\frac{7a_{10}}{11520}-\frac{7a_{11}}{46080}-\frac{7a_{13}}{46080}-\frac{a_{73}}{13824}-\frac{a_{75}}{4608}-\frac{a_{76}}{13824}\\&-\frac{a_{77}}{13824}-\frac{a_{79}}{4608}-\frac{a_{80}}{13824}-\frac{a_{81}}{13824}-\frac{a_{82}}{4608}-\frac{a_{83}}{13824}+\frac{31}{10752}.\end{aligned} \tag{59}$$

From the degree 0 part of $\tau_{3,0}\Phi(\tau_{0,0},\tau_{1,0})|_{t=0}=0$, we obtain

$$\begin{aligned}0=&\frac{7a_2}{144}+\frac{7a_3}{720}+\frac{7a_4}{720}+\frac{43a_9}{34560}+\frac{a_{10}}{480}+\frac{a_{11}}{1920}+\frac{a_{13}}{1920}+\frac{7a_{15}}{288}\\&+\frac{7a_{16}}{144}+\frac{7a_{17}}{720}+\frac{7a_{24}}{720}+\frac{7a_{28}}{2880}+\frac{7a_{30}}{2880}+\frac{a_{34}}{288}+\frac{a_{37}}{72}+\frac{a_{38}}{288}\\&+\frac{a_{42}}{288}+\frac{a_{44}}{72}+\frac{a_{45}}{288}+\frac{a_{53}}{72}+\frac{a_{54}}{288}+\frac{a_{55}}{288}+\frac{a_{57}}{72}+\frac{a_{58}}{288}\\&+\frac{a_{61}}{288}+\frac{a_{68}}{288}+\frac{a_{70}}{72}+\frac{a_{73}}{6912}+\frac{a_{75}}{2304}+\frac{a_{76}}{6912}+\frac{a_{77}}{6912}+\frac{a_{79}}{2304}\\&+\frac{a_{80}}{6912}+\frac{a_{83}}{6912}-\frac{859}{48384}.\end{aligned} \tag{60}$$

From the degree 0 part of $\tau_{2,0}\Phi(\tau_{0,0},\tau_{2,0})|_{t=0}=0$, we obtain

$$0=\frac{7a_2}{96}+\frac{7a_5}{2880}+\frac{7a_6}{960}+\frac{7a_7}{960}+\frac{31a_9}{23040}+\frac{221a_{10}}{69120}+\frac{7a_{11}}{23040}+\frac{7a_{15}}{288}$$

$$
\begin{aligned}
&+\frac{7a_{16}}{96}+\frac{7a_{24}}{480}+\frac{a_{32}}{288}+\frac{a_{33}}{288}+\frac{a_{37}}{48}+\frac{a_{39}}{288}+\frac{a_{40}}{288}+\frac{a_{44}}{48}\\
&+\frac{a_{47}}{288}+\frac{a_{53}}{48}+\frac{a_{57}}{48}+\frac{a_{59}}{288}+\frac{a_{60}}{288}+\frac{a_{62}}{288}+\frac{a_{65}}{288}+\frac{a_{66}}{288}\\
&+\frac{a_{67}}{288}+\frac{a_{70}}{48}+\frac{a_{73}}{6912}+\frac{a_{75}}{1152}+\frac{a_{77}}{6912}+\frac{a_{79}}{1152}+\frac{a_{81}}{6912}+\frac{a_{82}}{2304}\\
&+\frac{a_{83}}{6912}-\frac{859}{48384}. \qquad (61)
\end{aligned}
$$

From the degree 0 part of $\tau_{2,1}\Phi(\tau_{1,0},\tau_{0,0})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&-\frac{7a_{8}}{11520}-\frac{7a_{12}}{46080}-\frac{7a_{13}}{46080}-\frac{a_{72}}{13824}-\frac{a_{74}}{4608}-\frac{a_{76}}{13824}-\frac{a_{78}}{13824}\\
&-\frac{a_{79}}{4608}-\frac{a_{80}}{13824}-\frac{a_{81}}{13824}-\frac{a_{82}}{4608}-\frac{a_{83}}{13824}+\frac{31}{16128}. \qquad (62)
\end{aligned}
$$

From the degree 0 part of $\tau_{2,0}\Phi(\tau_{1,0},\tau_{1,0})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{7a_{2}}{48}+\frac{7a_{3}}{480}+\frac{7a_{4}}{240}+\frac{7a_{5}}{480}+\frac{7a_{6}}{240}+\frac{7a_{7}}{480}+\frac{a_{32}}{144}+\frac{a_{33}}{96}\\
&+\frac{a_{34}}{96}+\frac{a_{35}}{288}+\frac{a_{36}}{288}+\frac{a_{37}}{24}+\frac{a_{38}}{96}+\frac{a_{39}}{96}+\frac{a_{40}}{144}+\frac{a_{57}}{24}\\
&+\frac{a_{58}}{96}+\frac{a_{59}}{96}+\frac{a_{60}}{144}+\frac{a_{61}}{96}+\frac{a_{62}}{96}+\frac{a_{63}}{288}+\frac{a_{64}}{288}+\frac{a_{65}}{144}\\
&+\frac{a_{66}}{144}+\frac{a_{67}}{96}+\frac{a_{68}}{96}+\frac{a_{69}}{288}+\frac{a_{70}}{24}+\frac{5a_{79}}{3456}+\frac{a_{80}}{1728}+\frac{a_{81}}{1728}\\
&+\frac{a_{82}}{1152}+\frac{5a_{83}}{6912}-\frac{859}{48384}. \qquad (63)
\end{aligned}
$$

From the degree 1 part of $\tau_{5,1}\Phi(\tau_{0,0},\tau_{0,0})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{a_{1}}{30240}-\frac{a_{2}}{2880}-\frac{a_{8}}{11520}-\frac{a_{9}}{11520}-\frac{a_{10}}{11520}+\frac{7a_{14}}{2880}+\frac{a_{15}}{192}+\frac{7a_{16}}{2880}\\
&+\frac{7a_{18}}{2880}+\frac{a_{19}}{192}-\frac{a_{21}}{11520}-\frac{a_{22}}{11520}+\frac{a_{23}}{960}+\frac{a_{24}}{960}+\frac{a_{25}}{960}+\frac{a_{29}}{960}\\
&-\frac{a_{31}}{46080}-\frac{a_{37}}{288}-\frac{a_{43}}{288}-\frac{a_{44}}{288}-\frac{a_{50}}{288}-\frac{a_{52}}{288}-\frac{a_{53}}{288}-\frac{a_{57}}{288}\\
&+\frac{a_{70}}{288}+\frac{a_{74}}{13824}+\frac{a_{75}}{13824}+\frac{a_{79}}{13824}-\frac{a_{82}}{13824}+\frac{a_{84}}{6}+\frac{a_{88}}{6}+\frac{a_{89}}{6}\\
&+\frac{a_{90}}{6}+\frac{a_{92}}{12}-\frac{a_{103}}{6}+8a_{104}+\frac{a_{105}}{13824}-\frac{1}{5040}. \qquad (64)
\end{aligned}
$$

From the degree 1 part of $\tau_{4,1}\Phi(\tau_{0,0},\tau_{0,1})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{11a_{1}}{60480}+\frac{a_{2}}{960}+\frac{a_{8}}{34560}+\frac{a_{9}}{34560}-\frac{a_{10}}{11520}+\frac{19a_{14}}{2880}+\frac{a_{15}}{960}+\frac{a_{16}}{320}\\
&+\frac{a_{18}}{320}+\frac{a_{19}}{960}+\frac{a_{21}}{34560}-\frac{a_{22}}{11520}+\frac{7a_{23}}{2880}+\frac{a_{24}}{960}+\frac{a_{25}}{960}+\frac{a_{29}}{576}\\
&-\frac{a_{31}}{46080}-\frac{a_{37}}{288}-\frac{a_{44}}{288}-\frac{a_{50}}{288}-\frac{a_{52}}{288}-\frac{a_{53}}{288}-\frac{a_{57}}{288}+\frac{a_{70}}{288}\\
&+\frac{a_{74}}{13824}+\frac{a_{75}}{13824}+\frac{a_{79}}{13824}-\frac{a_{82}}{13824}+\frac{a_{84}}{6}+\frac{a_{88}}{6}+\frac{a_{89}}{12}+\frac{a_{90}}{6}\\
&+\frac{a_{92}}{8}-\frac{a_{93}}{24}-\frac{a_{103}}{6}+8a_{104}+\frac{a_{105}}{13824}-\frac{11}{20160}. \qquad (65)
\end{aligned}
$$

From the degree 1 part of $\tau_{5,0}\Phi(\tau_{0,0},\tau_{0,1})|_{t=0}=0$, we obtain

$$\begin{aligned}0=&-\frac{779a_1}{1451520}+\frac{a_2}{120}-\frac{13a_8}{34560}-\frac{13a_9}{34560}-\frac{13a_{10}}{34560}-\frac{a_{14}}{60}+\frac{a_{15}}{120}\\&+\frac{13a_{16}}{720}+\frac{13a_{18}}{720}+\frac{a_{19}}{120}-\frac{13a_{21}}{34560}-\frac{13a_{22}}{34560}-\frac{a_{23}}{120}+\frac{a_{24}}{180}+\frac{a_{25}}{180}\\&-\frac{a_{29}}{720}-\frac{13a_{31}}{138240}-\frac{a_{37}}{144}-\frac{a_{43}}{144}-\frac{a_{44}}{144}-\frac{a_{50}}{144}-\frac{a_{52}}{144}-\frac{a_{53}}{144}\\&-\frac{a_{57}}{144}+\frac{a_{74}}{13824}+\frac{a_{75}}{13824}+\frac{a_{79}}{13824}+\frac{a_{82}}{13824}+\frac{a_{84}}{2}+\frac{a_{88}}{2}+\frac{a_{89}}{6}\\&+\frac{a_{90}}{2}+\frac{a_{92}}{3}+\frac{a_{93}}{12}-\frac{a_{103}}{6}+16a_{104}+\frac{a_{105}}{13824}+\frac{61}{34560}.\end{aligned}\tag{66}$$

From the degree 1 part of $\tau_{4,1}\Phi(\tau_{0,0},\tau_{1,0})|_{t=0}=0$, we obtain

$$\begin{aligned}0=&-\frac{a_2}{144}+\frac{a_3}{240}-\frac{a_4}{720}-\frac{23a_9}{69120}-\frac{a_{10}}{3840}-\frac{a_{11}}{15360}-\frac{a_{13}}{15360}\\&+\frac{7a_{15}}{480}+\frac{a_{16}}{144}+\frac{a_{17}}{720}+\frac{a_{24}}{240}+\frac{a_{28}}{960}+\frac{a_{30}}{960}-\frac{a_{34}}{288}-\frac{a_{37}}{72}\\&-\frac{a_{38}}{288}-\frac{a_{42}}{288}-\frac{a_{44}}{72}-\frac{a_{45}}{288}-\frac{a_{53}}{72}-\frac{a_{54}}{288}-\frac{a_{55}}{288}-\frac{a_{57}}{72}\\&-\frac{a_{58}}{288}-\frac{a_{61}}{288}+\frac{a_{68}}{288}+\frac{a_{70}}{72}+\frac{a_{73}}{13824}+\frac{a_{75}}{4608}+\frac{a_{76}}{13824}+\frac{a_{77}}{13824}\\&+\frac{a_{79}}{4608}+\frac{a_{80}}{13824}-\frac{a_{81}}{13824}-\frac{a_{82}}{4608}+\frac{a_{83}}{13824}+\frac{5a_{84}}{6}+\frac{a_{85}}{6}+\frac{5a_{90}}{6}\\&+\frac{a_{91}}{6}+\frac{5a_{92}}{12}+\frac{a_{93}}{12}-\frac{a_{102}}{6}-\frac{5a_{103}}{6}+48a_{104}-\frac{41}{161280}.\end{aligned}\tag{67}$$

From the degree 1 part of $\tau_{3,1}\Phi(\tau_{0,0},\tau_{1,1})|_{t=0}=0$, we obtain

$$\begin{aligned}0=&-\frac{31a_1}{96768}+\frac{7a_3}{720}+\frac{7a_4}{720}-\frac{7a_8}{34560}-\frac{a_9}{13824}+\frac{7a_{11}}{138240}+\frac{7a_{13}}{138240}\\&+\frac{a_{15}}{288}+\frac{7a_{17}}{1440}-\frac{7a_{21}}{34560}-\frac{7a_{22}}{34560}+\frac{7a_{28}}{2880}+\frac{7a_{30}}{2880}-\frac{7a_{31}}{138240}-\frac{a_{34}}{288}\\&-\frac{a_{37}}{144}+\frac{a_{38}}{288}-\frac{a_{42}}{288}-\frac{a_{44}}{144}-\frac{a_{53}}{144}-\frac{a_{54}}{288}-\frac{a_{55}}{288}-\frac{a_{57}}{144}\\&+\frac{a_{68}}{288}+\frac{a_{70}}{72}+\frac{a_{73}}{13824}-\frac{a_{74}}{13824}+\frac{a_{75}}{6912}+\frac{a_{76}}{13824}+\frac{a_{77}}{13824}+\frac{a_{79}}{6912}\\&+\frac{a_{80}}{13824}-\frac{a_{81}}{13824}-\frac{a_{82}}{3456}+\frac{a_{83}}{13824}+\frac{a_{84}}{3}+\frac{a_{90}}{3}+\frac{a_{91}}{12}+\frac{a_{92}}{4}\\&+\frac{a_{93}}{24}-\frac{a_{95}}{24}-\frac{a_{97}}{12}-\frac{a_{102}}{6}-\frac{2a_{103}}{3}+32a_{104}-\frac{a_{105}}{13824}-\frac{1}{4608}.\end{aligned}\tag{68}$$

From the degree 1 part of $\tau_{4,0}\Phi(\tau_{0,0},\tau_{1,1})|_{t=0}=0$, we obtain

$$\begin{aligned}0=&\frac{2329a_1}{1451520}-\frac{a_2}{576}-\frac{a_3}{30}-\frac{a_4}{30}+\frac{a_8}{1440}-\frac{19a_9}{69120}-\frac{a_{10}}{2304}-\frac{13a_{11}}{46080}\\&-\frac{13a_{13}}{46080}+\frac{7a_{14}}{576}+\frac{11a_{15}}{576}+\frac{7a_{16}}{576}-\frac{11a_{17}}{720}+\frac{7a_{18}}{576}+\frac{7a_{19}}{576}+\frac{a_{21}}{1440}\\&+\frac{a_{22}}{1440}+\frac{7a_{23}}{2880}+\frac{a_{24}}{192}+\frac{7a_{25}}{2880}-\frac{a_{28}}{120}+\frac{7a_{29}}{2880}-\frac{a_{30}}{120}+\frac{a_{31}}{5760}\\&-\frac{a_{34}}{144}-\frac{7a_{37}}{288}-\frac{a_{38}}{144}-\frac{a_{42}}{144}+\frac{a_{43}}{288}-\frac{7a_{44}}{288}-\frac{a_{45}}{144}+\frac{a_{50}}{288}\end{aligned}$$

$$
\begin{aligned}
&+ \frac{a_{52}}{288} - \frac{7a_{53}}{288} - \frac{a_{54}}{144} - \frac{a_{55}}{144} - \frac{7a_{57}}{288} - \frac{a_{58}}{144} - \frac{a_{61}}{144} + \frac{a_{70}}{288} \\
&+ \frac{a_{73}}{13824} + \frac{a_{74}}{6912} + \frac{5a_{75}}{13824} + \frac{a_{76}}{13824} + \frac{a_{77}}{13824} + \frac{5a_{79}}{13824} + \frac{a_{80}}{13824} + \frac{a_{81}}{13824} \\
&+ \frac{a_{82}}{4608} + \frac{a_{83}}{13824} + \frac{3a_{84}}{2} - \frac{a_{85}}{6} + \frac{3a_{90}}{2} + \frac{a_{91}}{6} + \frac{11a_{92}}{12} + \frac{a_{95}}{12} \\
&+ \frac{a_{97}}{6} - \frac{a_{102}}{6} - \frac{5a_{103}}{6} + 72a_{104} + \frac{a_{105}}{6912} - \frac{131}{69120}. \qquad (69)
\end{aligned}
$$

From the degree 1 part of $\tau_{3,1}\Phi(\tau_{0,0},\tau_{2,0})|_{t=0} = 0$, we obtain

$$
\begin{aligned}
0 = &\frac{31a_1}{96768} - \frac{a_2}{288} + \frac{7a_5}{2880} + \frac{7a_6}{960} - \frac{7a_7}{2880} + \frac{7a_8}{34560} - \frac{a_9}{2304} - \frac{a_{10}}{1152} \\
&- \frac{7a_{11}}{69120} + \frac{7a_{15}}{288} + \frac{7a_{16}}{288} + \frac{7a_{21}}{34560} + \frac{7a_{22}}{34560} + \frac{a_{24}}{96} + \frac{7a_{31}}{138240} - \frac{a_{32}}{288} \\
&- \frac{a_{33}}{288} + \frac{a_{34}}{144} - \frac{a_{37}}{36} + \frac{a_{39}}{288} - \frac{a_{40}}{288} + \frac{a_{42}}{144} - \frac{a_{44}}{36} - \frac{a_{47}}{288} \\
&- \frac{a_{53}}{36} + \frac{a_{55}}{144} - \frac{a_{57}}{36} + \frac{a_{59}}{288} - \frac{a_{60}}{288} - \frac{a_{62}}{288} - \frac{a_{65}}{288} + \frac{a_{66}}{288} \\
&- \frac{a_{67}}{288} - \frac{a_{68}}{144} + \frac{a_{70}}{48} - \frac{a_{73}}{6912} + \frac{a_{74}}{13824} + \frac{7a_{75}}{13824} - \frac{a_{77}}{6912} + \frac{7a_{79}}{13824} \\
&+ \frac{a_{81}}{6912} + \frac{a_{82}}{13824} - \frac{a_{83}}{6912} + \frac{5a_{84}}{3} + \frac{a_{87}}{6} + \frac{5a_{90}}{3} + \frac{5a_{92}}{6} + \frac{a_{95}}{12} \\
&- \frac{a_{98}}{6} - \frac{a_{100}}{6} + \frac{a_{102}}{3} - \frac{5a_{103}}{3} + 112a_{104} + \frac{a_{105}}{13824} - \frac{53}{48384}. \qquad (70)
\end{aligned}
$$

From the degree 1 part of $\tau_{2,1}\Phi(\tau_{0,0},\tau_{2,1})|_{t=0} = 0$, we obtain

$$
\begin{aligned}
0 = &\frac{a_2}{288} + \frac{7a_5}{2880} + \frac{7a_6}{960} + \frac{7a_7}{960} - \frac{a_9}{13824} - \frac{a_{10}}{3456} + \frac{7a_{11}}{46080} - \frac{7a_{13}}{46080} \\
&+ \frac{a_{15}}{288} + \frac{a_{16}}{96} + \frac{a_{24}}{288} - \frac{a_{32}}{288} - \frac{a_{33}}{288} - \frac{a_{37}}{144} + \frac{a_{39}}{288} + \frac{a_{40}}{288} \\
&- \frac{a_{44}}{144} - \frac{a_{47}}{288} - \frac{a_{53}}{144} - \frac{a_{57}}{144} + \frac{a_{66}}{288} + \frac{a_{67}}{288} + \frac{a_{70}}{48} + \frac{a_{73}}{13824} \\
&+ \frac{a_{75}}{4608} - \frac{a_{76}}{13824} + \frac{a_{77}}{13824} + \frac{a_{79}}{4608} - \frac{a_{80}}{13824} + \frac{a_{81}}{13824} - \frac{a_{82}}{4608} + \frac{a_{83}}{13824} \\
&+ \frac{a_{84}}{3} + \frac{a_{90}}{3} + \frac{a_{92}}{4} + \frac{a_{94}}{24} + \frac{a_{95}}{24} - \frac{a_{98}}{12} - \frac{a_{99}}{12} - \frac{a_{100}}{6} \\
&- \frac{a_{101}}{6} - a_{103} + 48a_{104} - \frac{1}{4608}. \qquad (71)
\end{aligned}
$$

From the degree 1 part of $\tau_{3,0}\Phi(\tau_{0,0},\tau_{2,1})|_{t=0} = 0$, we obtain

$$
\begin{aligned}
0 = &-\frac{31a_1}{96768} + \frac{7a_3}{720} + \frac{7a_4}{720} - \frac{a_7}{40} - \frac{7a_8}{34560} - \frac{23a_9}{69120} - \frac{13a_{10}}{23040} \\
&- \frac{a_{11}}{1920} + \frac{a_{13}}{1920} + \frac{7a_{15}}{288} + \frac{5a_{16}}{144} + \frac{7a_{17}}{720} - \frac{7a_{21}}{34560} - \frac{7a_{22}}{34560} + \frac{a_{24}}{72} \\
&+ \frac{7a_{28}}{2880} + \frac{7a_{30}}{2880} - \frac{7a_{31}}{138240} - \frac{a_{32}}{144} + \frac{a_{33}}{144} - \frac{a_{34}}{288} - \frac{5a_{37}}{144} + \frac{a_{38}}{288} \\
&- \frac{a_{40}}{144} - \frac{a_{42}}{288} - \frac{5a_{44}}{144} + \frac{a_{45}}{288} - \frac{5a_{53}}{144} + \frac{a_{54}}{288} - \frac{a_{55}}{288} - \frac{5a_{57}}{144} \\
&+ \frac{a_{58}}{288} - \frac{a_{60}}{144} - \frac{a_{61}}{288} - \frac{a_{65}}{144} - \frac{a_{67}}{144} - \frac{a_{68}}{288} + \frac{a_{70}}{72} - \frac{a_{73}}{6912}
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{a_{74}}{13824} + \frac{11a_{75}}{13824} + \frac{a_{76}}{6912} - \frac{a_{77}}{6912} + \frac{11a_{79}}{13824} + \frac{a_{80}}{6912} + \frac{5a_{82}}{13824} - \frac{a_{83}}{6912} \\
&+ 2a_{84} - \frac{a_{87}}{6} + 2a_{90} + \frac{13a_{92}}{12} - \frac{a_{97}}{6} + \frac{a_{99}}{6} - \frac{a_{100}}{6} + \frac{a_{101}}{3} \\
&- \frac{5a_{103}}{3} + 128a_{104} - \frac{a_{105}}{13824} - \frac{53}{48384}. \qquad (72)
\end{aligned}
$$

From the degree 1 part of $\tau_{2,1}\Phi(\tau_{0,0}, \tau_{3,0})|_{t=0} = 0$, we obtain

$$
\begin{aligned}
0 = &\frac{7a_2}{720} - \frac{a_5}{120} - \frac{a_6}{40} - \frac{a_9}{3072} - \frac{113a_{10}}{138240} + \frac{7a_{11}}{46080} + \frac{7a_{13}}{46080} + \frac{31a_{15}}{1440} \\
&+ \frac{2a_{16}}{45} + \frac{3a_{24}}{160} + \frac{a_{32}}{144} - \frac{a_{33}}{144} - \frac{5a_{37}}{144} - \frac{a_{39}}{144} - \frac{5a_{44}}{144} - \frac{5a_{53}}{144} \\
&- \frac{5a_{57}}{144} - \frac{a_{59}}{144} - \frac{a_{62}}{144} - \frac{a_{66}}{144} + \frac{a_{70}}{144} + \frac{a_{73}}{13824} + \frac{a_{75}}{2304} + \frac{a_{76}}{13824} \\
&+ \frac{a_{77}}{13824} + \frac{a_{79}}{2304} + \frac{a_{80}}{13824} + \frac{a_{81}}{13824} + \frac{a_{82}}{6912} + \frac{a_{83}}{13824} + 2a_{84} - \frac{a_{86}}{6} \\
&+ 2a_{90} + \frac{13a_{92}}{12} - \frac{a_{96}}{6} + \frac{a_{98}}{6} + \frac{a_{100}}{3} - \frac{a_{101}}{6} - \frac{5a_{103}}{3} + 128a_{104} \\
&- \frac{127}{80640}. \qquad (73)
\end{aligned}
$$

From the degree 1 part of $\tau_{2,0}\Phi(\tau_{0,0}, \tau_{3,1})|_{t=0} = 0$, we obtain

$$
\begin{aligned}
0 = &-\frac{a_2}{288} + \frac{a_5}{960} - \frac{7a_6}{2880} + \frac{7a_7}{960} - \frac{11a_9}{46080} - \frac{47a_{10}}{46080} + \frac{7a_{11}}{46080} \\
&- \frac{7a_{13}}{46080} + \frac{7a_{15}}{480} + \frac{7a_{16}}{288} + \frac{a_{24}}{96} - \frac{a_{32}}{288} - \frac{a_{33}}{288} - \frac{a_{37}}{48} - \frac{a_{39}}{288} \\
&+ \frac{a_{40}}{288} - \frac{a_{44}}{48} - \frac{a_{47}}{288} - \frac{a_{53}}{48} - \frac{a_{57}}{36} - \frac{a_{59}}{288} + \frac{a_{60}}{288} - \frac{a_{62}}{288} \\
&- \frac{a_{65}}{288} - \frac{a_{66}}{288} + \frac{a_{67}}{288} + \frac{a_{70}}{144} + \frac{a_{73}}{13824} + \frac{a_{75}}{6912} - \frac{a_{76}}{13824} + \frac{a_{77}}{13824} \\
&+ \frac{a_{79}}{6912} - \frac{a_{80}}{13824} + \frac{a_{81}}{13824} + \frac{a_{82}}{3456} + \frac{a_{83}}{13824} + \frac{5a_{84}}{3} + \frac{a_{86}}{6} + \frac{5a_{90}}{3} \\
&+ \frac{5a_{92}}{6} + \frac{a_{94}}{12} - \frac{a_{99}}{6} - \frac{a_{101}}{6} - \frac{4a_{103}}{3} + 112a_{104} - \frac{139}{241920}. \qquad (74)
\end{aligned}
$$

From the degree 1 part of $\tau_{4,1}\Phi(\tau_{0,1}, \tau_{0,0})|_{t=0} = 0$, we obtain

$$
\begin{aligned}
0 = &\frac{11a_1}{60480} + \frac{a_2}{960} - \frac{a_8}{11520} - \frac{a_9}{11520} + \frac{a_{10}}{34560} + \frac{a_{14}}{320} + \frac{5a_{15}}{576} + \frac{19a_{16}}{2880} \\
&+ \frac{a_{18}}{320} + \frac{a_{19}}{960} + \frac{a_{21}}{34560} - \frac{a_{22}}{11520} + \frac{a_{23}}{960} + \frac{7a_{24}}{2880} + \frac{a_{25}}{960} + \frac{a_{29}}{576} \\
&- \frac{a_{31}}{46080} - \frac{a_{37}}{288} - \frac{a_{43}}{288} - \frac{a_{50}}{288} - \frac{a_{52}}{288} - \frac{a_{53}}{288} - \frac{a_{57}}{288} + \frac{a_{70}}{288} \\
&+ \frac{a_{74}}{13824} + \frac{a_{75}}{13824} + \frac{a_{79}}{13824} - \frac{a_{82}}{13824} + \frac{a_{84}}{6} + \frac{a_{88}}{6} + \frac{a_{89}}{6} + \frac{a_{90}}{12} \\
&+ \frac{a_{92}}{8} - \frac{a_{94}}{24} - \frac{a_{103}}{6} + 8a_{104} + \frac{a_{105}}{13824} - \frac{11}{10080}. \qquad (75)
\end{aligned}
$$

From the degree 1 part of $\tau_{5,0}\Phi(\tau_{0,1}, \tau_{0,0})|_{t=0} = 0$, we obtain

$$
0 = -\frac{779a_1}{1451520} + \frac{a_2}{120} - \frac{13a_8}{34560} - \frac{13a_9}{34560} - \frac{13a_{10}}{34560} + \frac{13a_{14}}{720} - \frac{a_{15}}{144}
$$

$$
\begin{aligned}
&-\frac{a_{16}}{60}+\frac{13a_{18}}{720}+\frac{a_{19}}{120}-\frac{13a_{21}}{34560}-\frac{13a_{22}}{34560}+\frac{a_{23}}{180}-\frac{a_{24}}{120}+\frac{a_{25}}{180}\\
&-\frac{a_{29}}{720}-\frac{13a_{31}}{138240}-\frac{a_{37}}{144}-\frac{a_{43}}{144}-\frac{a_{44}}{144}-\frac{a_{50}}{144}-\frac{a_{52}}{144}-\frac{a_{53}}{144}\\
&-\frac{a_{57}}{144}+\frac{a_{74}}{13824}+\frac{a_{75}}{13824}+\frac{a_{79}}{13824}+\frac{a_{82}}{13824}+\frac{a_{84}}{2}+\frac{a_{88}}{2}+\frac{a_{89}}{2}\\
&+\frac{a_{90}}{6}+\frac{a_{92}}{3}+\frac{a_{94}}{12}-\frac{a_{103}}{6}+16a_{104}+\frac{a_{105}}{13824}+\frac{779}{241920}.
\end{aligned}\tag{76}
$$

From the degree 1 part of $\tau_{3,1}\Phi(\tau_{0,1},\tau_{0,1})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&-\frac{31a_{1}}{96768}+\frac{7a_{17}}{1440}-\frac{7a_{21}}{34560}-\frac{7a_{22}}{34560}+\frac{7a_{27}}{5760}+\frac{7a_{28}}{5760}+\frac{7a_{30}}{2880}\\
&-\frac{7a_{31}}{138240}-\frac{a_{37}}{288}-\frac{a_{43}}{576}-\frac{a_{44}}{576}+\frac{a_{45}}{576}-\frac{a_{52}}{288}-\frac{a_{53}}{288}-\frac{a_{57}}{288}\\
&+\frac{a_{70}}{288}+\frac{a_{74}}{13824}+\frac{a_{75}}{13824}+\frac{a_{79}}{13824}-\frac{a_{82}}{13824}+\frac{a_{84}}{6}+\frac{a_{89}}{12}+\frac{a_{90}}{12}\\
&+\frac{a_{92}}{8}-\frac{a_{95}}{24}-\frac{a_{103}}{6}+8a_{104}-\frac{a_{105}}{13824}.
\end{aligned}\tag{77}
$$

From the degree 1 part of $\tau_{4,0}\Phi(\tau_{0,1},\tau_{0,1})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{2329a_{1}}{1451520}+\frac{a_{2}}{192}-\frac{a_{8}}{17280}-\frac{a_{9}}{17280}-\frac{a_{10}}{17280}+\frac{5a_{14}}{576}+\frac{5a_{15}}{576}+\frac{5a_{16}}{576}\\
&-\frac{a_{17}}{60}+\frac{7a_{18}}{576}+\frac{7a_{19}}{576}+\frac{a_{21}}{1440}+\frac{a_{22}}{1440}+\frac{7a_{23}}{2880}+\frac{7a_{24}}{2880}+\frac{7a_{25}}{2880}\\
&-\frac{a_{27}}{240}-\frac{a_{28}}{240}+\frac{7a_{29}}{2880}-\frac{a_{30}}{120}+\frac{a_{31}}{5760}-\frac{a_{37}}{96}-\frac{a_{43}}{288}-\frac{a_{44}}{288}\\
&-\frac{a_{45}}{288}+\frac{a_{50}}{288}-\frac{a_{52}}{96}-\frac{a_{53}}{96}-\frac{a_{57}}{96}+\frac{a_{70}}{288}+\frac{a_{74}}{6912}+\frac{a_{75}}{6912}\\
&+\frac{a_{79}}{6912}+\frac{2a_{84}}{3}+\frac{a_{89}}{3}+\frac{a_{90}}{3}-\frac{a_{91}}{12}+\frac{a_{92}}{2}-\frac{a_{93}}{24}-\frac{a_{94}}{24}\\
&+\frac{a_{95}}{12}-\frac{a_{103}}{3}+24a_{104}+\frac{a_{105}}{6912}-\frac{25}{16128}.
\end{aligned}\tag{78}
$$

From the degree 1 part of $\tau_{3,1}\Phi(\tau_{0,1},\tau_{1,0})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&-\frac{a_{9}}{4608}+\frac{7a_{10}}{34560}+\frac{7a_{11}}{138240}+\frac{7a_{13}}{138240}+\frac{5a_{15}}{288}+\frac{7a_{16}}{288}+\frac{7a_{17}}{1440}\\
&+\frac{7a_{24}}{720}+\frac{7a_{28}}{2880}+\frac{7a_{30}}{2880}-\frac{a_{34}}{288}-\frac{a_{37}}{72}-\frac{a_{38}}{288}-\frac{a_{53}}{72}-\frac{a_{54}}{288}\\
&-\frac{a_{55}}{288}-\frac{a_{57}}{72}-\frac{a_{58}}{288}-\frac{a_{61}}{288}+\frac{a_{68}}{288}+\frac{a_{70}}{72}+\frac{a_{73}}{13824}+\frac{a_{75}}{4608}\\
&+\frac{a_{76}}{13824}+\frac{a_{77}}{13824}+\frac{a_{79}}{4608}+\frac{a_{80}}{13824}-\frac{a_{81}}{13824}-\frac{a_{82}}{4608}+\frac{a_{83}}{13824}+\frac{5a_{84}}{6}\\
&+\frac{a_{85}}{6}+\frac{5a_{90}}{12}+\frac{a_{91}}{12}+\frac{5a_{92}}{8}+\frac{a_{93}}{8}-\frac{a_{94}}{6}-\frac{a_{95}}{12}-\frac{a_{102}}{6}\\
&-\frac{5a_{103}}{6}+48a_{104}-\frac{23}{10752}.
\end{aligned}\tag{79}
$$

From the degree 1 part of $\tau_{4,0}\Phi(\tau_{0,1},\tau_{1,0})|_{t=0}=0$, we obtain

$$
0=\frac{a_{2}}{72}+\frac{a_{3}}{360}+\frac{a_{4}}{360}-\frac{67a_{9}}{69120}-\frac{13a_{10}}{11520}-\frac{13a_{11}}{46080}-\frac{13a_{13}}{46080}-\frac{a_{15}}{60}
$$

$$
\begin{aligned}
&-\frac{11a_{16}}{144}-\frac{11a_{17}}{720}-\frac{a_{24}}{30}-\frac{a_{28}}{120}-\frac{a_{30}}{120}-\frac{a_{34}}{144}-\frac{a_{37}}{36}-\frac{a_{38}}{144}\\
&-\frac{a_{42}}{144}-\frac{a_{44}}{36}-\frac{a_{45}}{144}-\frac{a_{53}}{36}-\frac{a_{54}}{144}-\frac{a_{55}}{144}-\frac{a_{57}}{36}-\frac{a_{58}}{144}\\
&-\frac{a_{61}}{144}+\frac{a_{73}}{13824}+\frac{a_{75}}{4608}+\frac{a_{76}}{13824}+\frac{a_{77}}{13824}+\frac{a_{79}}{4608}+\frac{a_{80}}{13824}+\frac{a_{81}}{13824}\\
&+\frac{a_{82}}{4608}+\frac{a_{83}}{13824}+\frac{5a_{84}}{2}+\frac{a_{85}}{2}+\frac{5a_{90}}{6}+\frac{a_{91}}{6}+\frac{5a_{92}}{3}+\frac{a_{93}}{3}\\
&+\frac{a_{94}}{3}+\frac{a_{95}}{6}-\frac{a_{102}}{6}-\frac{5a_{103}}{6}+96a_{104}+\frac{1247}{161280}.
\end{aligned}
\tag{80}
$$

From the degree 1 part of $\tau_{2,1}\Phi(\tau_{0,1},\tau_{1,1})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{7a_7}{960}-\frac{a_9}{13824}-\frac{7a_{11}}{46080}-\frac{7a_{13}}{46080}+\frac{a_{15}}{576}-\frac{a_{34}}{288}-\frac{a_{37}}{288}+\frac{a_{40}}{288}\\
&-\frac{a_{42}}{576}-\frac{a_{44}}{576}-\frac{a_{53}}{288}-\frac{a_{57}}{288}-\frac{a_{58}}{576}+\frac{a_{60}}{576}-\frac{a_{61}}{576}+\frac{a_{63}}{576}\\
&+\frac{a_{68}}{288}+\frac{a_{70}}{96}+\frac{a_{73}}{13824}+\frac{a_{75}}{13824}-\frac{a_{76}}{13824}-\frac{a_{77}}{13824}+\frac{a_{79}}{13824}+\frac{a_{80}}{13824}\\
&-\frac{a_{81}}{13824}-\frac{a_{82}}{4608}+\frac{a_{83}}{13824}+\frac{a_{84}}{6}+\frac{a_{90}}{12}+\frac{a_{92}}{8}+\frac{a_{93}}{24}-\frac{a_{99}}{12}\\
&-\frac{a_{102}}{6}-\frac{a_{103}}{2}+24a_{104}-\frac{1}{13824}.
\end{aligned}
\tag{81}
$$

From the degree 1 part of $\tau_{3,0}\Phi(\tau_{0,1},\tau_{1,1})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&-\frac{31a_1}{96768}+\frac{7a_3}{720}+\frac{7a_4}{720}-\frac{a_7}{40}-\frac{7a_8}{34560}-\frac{a_9}{3456}+\frac{7a_{10}}{34560}\\
&+\frac{a_{11}}{1920}+\frac{a_{13}}{1920}+\frac{a_{15}}{48}+\frac{7a_{16}}{288}+\frac{7a_{17}}{720}-\frac{7a_{21}}{34560}-\frac{7a_{22}}{34560}+\frac{7a_{24}}{720}\\
&+\frac{7a_{28}}{2880}+\frac{7a_{30}}{2880}-\frac{7a_{31}}{138240}-\frac{a_{34}}{96}-\frac{a_{37}}{48}+\frac{a_{38}}{288}-\frac{a_{40}}{144}-\frac{a_{42}}{288}\\
&-\frac{a_{44}}{288}+\frac{a_{45}}{288}-\frac{a_{53}}{48}+\frac{a_{54}}{288}+\frac{a_{55}}{288}-\frac{a_{57}}{48}-\frac{a_{58}}{288}-\frac{a_{60}}{288}\\
&-\frac{a_{61}}{288}-\frac{a_{63}}{288}+\frac{a_{68}}{288}+\frac{a_{70}}{72}+\frac{a_{73}}{6912}-\frac{a_{74}}{13824}+\frac{5a_{75}}{13824}+\frac{a_{76}}{6912}\\
&+\frac{a_{77}}{6912}+\frac{5a_{79}}{13824}+\frac{a_{80}}{6912}-\frac{a_{82}}{13824}+\frac{a_{83}}{6912}+\frac{7a_{84}}{6}-\frac{a_{87}}{6}+\frac{7a_{90}}{12}\\
&+\frac{7a_{92}}{8}+\frac{a_{93}}{6}-\frac{a_{94}}{6}-\frac{a_{95}}{8}-\frac{a_{97}}{12}+\frac{a_{99}}{6}-\frac{a_{102}}{3}-\frac{7a_{103}}{6}\\
&+80a_{104}-\frac{a_{105}}{13824}-\frac{19}{8064}.
\end{aligned}
\tag{82}
$$

From the degree 1 part of $\tau_{2,1}\Phi(\tau_{0,1},\tau_{2,0})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{a_2}{96}+\frac{7a_5}{2880}+\frac{7a_6}{960}+\frac{7a_7}{960}-\frac{a_9}{3456}+\frac{a_{10}}{11520}+\frac{7a_{11}}{23040}+\frac{a_{15}}{48}\\
&+\frac{a_{16}}{24}+\frac{7a_{24}}{480}-\frac{a_{32}}{288}-\frac{a_{33}}{288}+\frac{a_{34}}{144}-\frac{a_{37}}{48}+\frac{a_{39}}{288}+\frac{a_{40}}{288}\\
&+\frac{a_{42}}{288}-\frac{a_{53}}{48}-\frac{a_{57}}{48}+\frac{a_{59}}{288}+\frac{a_{61}}{288}-\frac{a_{62}}{288}-\frac{a_{63}}{288}+\frac{a_{66}}{288}\\
&-\frac{a_{67}}{288}-\frac{a_{68}}{144}+\frac{a_{70}}{48}-\frac{a_{73}}{6912}+\frac{a_{75}}{2304}+\frac{a_{77}}{6912}+\frac{a_{79}}{2304}+\frac{a_{81}}{6912}
\end{aligned}
$$

$$- \frac{a_{83}}{6912} + \frac{7a_{84}}{6} + \frac{7a_{90}}{12} + \frac{7a_{92}}{8} - \frac{a_{94}}{4} + \frac{a_{95}}{24} + \frac{a_{97}}{12} - \frac{a_{98}}{6}$$
$$- \frac{a_{99}}{6} - \frac{a_{100}}{6} + \frac{a_{102}}{3} - \frac{3a_{103}}{2} + 96a_{104} - \frac{19}{8064}. \quad (83)$$

From the degree 1 part of $\tau_{3,0}\Phi(\tau_{0,1}, \tau_{2,0})|_{t=0} = 0$, we obtain

$$0 = \frac{31a_1}{96768} + \frac{7a_2}{144} - \frac{a_7}{40} + \frac{7a_8}{34560} - \frac{3a_9}{2560} - \frac{31a_{10}}{13824} - \frac{a_{11}}{960} - \frac{11a_{15}}{720}$$
$$- \frac{a_{16}}{8} + \frac{7a_{21}}{34560} + \frac{7a_{22}}{34560} - \frac{43a_{24}}{720} + \frac{7a_{31}}{138240} - \frac{a_{32}}{144} - \frac{a_{33}}{144} + \frac{a_{34}}{48}$$
$$- \frac{7a_{37}}{144} - \frac{a_{40}}{144} + \frac{a_{42}}{144} - \frac{7a_{44}}{144} - \frac{a_{47}}{144} - \frac{7a_{53}}{144} - \frac{a_{55}}{144} - \frac{7a_{57}}{144}$$
$$- \frac{a_{60}}{144} + \frac{a_{61}}{144} - \frac{a_{62}}{144} + \frac{a_{63}}{144} - \frac{a_{65}}{144} - \frac{a_{67}}{144} - \frac{a_{68}}{144} - \frac{a_{73}}{3456}$$
$$+ \frac{a_{74}}{13824} + \frac{7a_{75}}{13824} - \frac{a_{77}}{3456} + \frac{7a_{79}}{13824} + \frac{7a_{82}}{13824} - \frac{a_{83}}{3456} + \frac{13a_{84}}{3} - \frac{a_{87}}{6}$$
$$+ \frac{4a_{90}}{3} + \frac{17a_{92}}{6} + \frac{7a_{94}}{12} + \frac{a_{97}}{6} - \frac{a_{98}}{6} + \frac{a_{99}}{3} - \frac{a_{100}}{6} + \frac{2a_{102}}{3}$$
$$- \frac{5a_{103}}{3} + 208a_{104} + \frac{a_{105}}{13824} + \frac{139}{15120}. \quad (84)$$

From the degree 1 part of $\tau_{2,0}\Phi(\tau_{0,1}, \tau_{2,1})|_{t=0} = 0$, we obtain

$$0 = \frac{a_2}{96} + \frac{7a_5}{2880} + \frac{7a_6}{960} + \frac{7a_7}{960} - \frac{a_9}{8640} + \frac{a_{10}}{2560} + \frac{7a_{11}}{46080} - \frac{7a_{13}}{46080}$$
$$+ \frac{5a_{15}}{288} + \frac{a_{16}}{24} + \frac{7a_{24}}{480} - \frac{a_{32}}{288} - \frac{a_{33}}{288} - \frac{a_{37}}{72} + \frac{a_{39}}{288} + \frac{a_{40}}{288}$$
$$+ \frac{a_{44}}{288} - \frac{a_{53}}{48} - \frac{5a_{57}}{288} - \frac{a_{59}}{288} + \frac{a_{60}}{288} - \frac{a_{65}}{288} - \frac{a_{66}}{288} + \frac{a_{67}}{288}$$
$$+ \frac{a_{70}}{144} + \frac{a_{73}}{13824} + \frac{5a_{75}}{13824} - \frac{a_{76}}{13824} + \frac{a_{77}}{13824} + \frac{a_{79}}{13824} - \frac{a_{80}}{13824} + \frac{a_{81}}{13824}$$
$$+ \frac{a_{82}}{4608} + \frac{a_{83}}{13824} + \frac{7a_{84}}{6} + \frac{7a_{90}}{12} + \frac{7a_{92}}{8} - \frac{5a_{94}}{24} + \frac{a_{96}}{12} - \frac{a_{98}}{6}$$
$$- \frac{a_{99}}{6} - \frac{a_{101}}{6} - \frac{7a_{103}}{6} + 96a_{104} - \frac{23}{10752}. \quad (85)$$

From the degree 1 part of $\tau_{2,0}\Phi(\tau_{0,1}, \tau_{3,0})|_{t=0} = 0$, we obtain

$$0 = \frac{7a_2}{144} - \frac{a_5}{120} - \frac{a_6}{40} - \frac{35a_9}{27648} - \frac{503a_{10}}{138240} + \frac{7a_{11}}{46080} + \frac{7a_{13}}{46080} - \frac{a_{15}}{60}$$
$$- \frac{a_{16}}{8} - \frac{43a_{24}}{720} - \frac{a_{32}}{144} - \frac{a_{33}}{144} - \frac{a_{37}}{36} - \frac{a_{39}}{144} - \frac{a_{44}}{24} - \frac{a_{47}}{144}$$
$$- \frac{a_{53}}{18} - \frac{a_{57}}{24} - \frac{a_{62}}{144} - \frac{a_{65}}{144} - \frac{a_{66}}{144} - \frac{a_{70}}{72} + \frac{a_{73}}{13824} - \frac{a_{75}}{3456}$$
$$+ \frac{a_{76}}{13824} + \frac{a_{77}}{13824} - \frac{a_{79}}{3456} + \frac{a_{80}}{13824} + \frac{a_{81}}{13824} + \frac{a_{82}}{3456} + \frac{a_{83}}{13824} + \frac{13a_{84}}{3}$$
$$- \frac{a_{86}}{6} + \frac{4a_{90}}{3} + \frac{17a_{92}}{6} + \frac{7a_{94}}{12} + \frac{a_{96}}{6} + \frac{a_{98}}{3} - \frac{a_{99}}{6} - \frac{a_{101}}{6}$$
$$- a_{103} + 208a_{104} + \frac{1793}{241920}. \quad (86)$$

From the degree 1 part of $\tau_{4,1}\Phi(\tau_{1,0},\tau_{0,0})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0 = &-\frac{a_2}{144}+\frac{a_5}{240}-\frac{a_6}{720}-\frac{a_8}{3840}-\frac{a_{12}}{15360}-\frac{a_{13}}{15360}+\frac{a_{14}}{144}\\
&+\frac{a_{17}}{720}+\frac{a_{23}}{240}+\frac{a_{27}}{960}+\frac{a_{28}}{960}+\frac{a_{30}}{960}-\frac{a_{33}}{288}-\frac{a_{37}}{72}-\frac{a_{39}}{288}\\
&-\frac{a_{41}}{288}-\frac{a_{43}}{72}-\frac{a_{45}}{288}-\frac{a_{52}}{72}-\frac{a_{54}}{288}-\frac{a_{56}}{288}-\frac{a_{57}}{72}-\frac{a_{59}}{288}\\
&-\frac{a_{62}}{288}+\frac{a_{67}}{288}+\frac{a_{70}}{72}+\frac{a_{72}}{13824}+\frac{a_{74}}{4608}+\frac{a_{76}}{13824}+\frac{a_{78}}{13824}+\frac{a_{79}}{4608}\\
&-\frac{a_{80}}{13824}+\frac{a_{81}}{13824}-\frac{a_{82}}{4608}+\frac{a_{83}}{13824}+\frac{5a_{84}}{6}+\frac{a_{86}}{6}+\frac{5a_{89}}{6}+\frac{a_{91}}{6}\\
&+\frac{5a_{92}}{12}+\frac{a_{94}}{12}-\frac{a_{101}}{6}-\frac{5a_{103}}{6}+48a_{104}+\frac{5}{16128}.
\end{aligned} \tag{87}
$$

From the degree 1 part of $\tau_{3,1}\Phi(\tau_{1,0},\tau_{0,1})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0 = &\frac{7a_8}{34560}+\frac{7a_{12}}{138240}+\frac{7a_{13}}{138240}+\frac{7a_{14}}{288}+\frac{7a_{17}}{1440}+\frac{7a_{23}}{720}+\frac{7a_{27}}{2880}+\frac{7a_{28}}{2880}\\
&+\frac{7a_{30}}{2880}-\frac{a_{33}}{288}-\frac{a_{37}}{72}-\frac{a_{39}}{288}-\frac{a_{52}}{72}-\frac{a_{54}}{288}-\frac{a_{56}}{288}-\frac{a_{57}}{72}\\
&-\frac{a_{59}}{288}-\frac{a_{62}}{288}+\frac{a_{67}}{288}+\frac{a_{70}}{72}+\frac{a_{72}}{13824}+\frac{a_{74}}{4608}+\frac{a_{76}}{13824}+\frac{a_{78}}{13824}\\
&+\frac{a_{79}}{4608}-\frac{a_{80}}{13824}+\frac{a_{81}}{13824}-\frac{a_{82}}{4608}+\frac{a_{83}}{13824}+\frac{5a_{84}}{6}+\frac{a_{86}}{6}+\frac{5a_{89}}{12}\\
&+\frac{a_{91}}{12}+\frac{5a_{92}}{8}-\frac{a_{93}}{6}+\frac{a_{94}}{8}-\frac{a_{95}}{12}-\frac{a_{101}}{6}-\frac{5a_{103}}{6}+48a_{104}\\
&-\frac{31}{48384}.
\end{aligned} \tag{88}
$$

From the degree 1 part of $\tau_{4,0}\Phi(\tau_{1,0},\tau_{0,1})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0 = &\frac{a_2}{72}+\frac{a_5}{360}+\frac{a_6}{360}-\frac{13a_8}{11520}-\frac{13a_{12}}{46080}-\frac{13a_{13}}{46080}-\frac{11a_{14}}{144}-\frac{11a_{17}}{720}\\
&-\frac{a_{23}}{30}-\frac{a_{27}}{120}-\frac{a_{28}}{120}-\frac{a_{30}}{120}-\frac{a_{33}}{144}-\frac{a_{37}}{36}-\frac{a_{39}}{144}-\frac{a_{41}}{144}\\
&-\frac{a_{43}}{36}-\frac{a_{45}}{144}-\frac{a_{52}}{36}-\frac{a_{54}}{144}-\frac{a_{56}}{144}-\frac{a_{57}}{36}-\frac{a_{59}}{144}-\frac{a_{62}}{144}\\
&+\frac{a_{72}}{13824}+\frac{a_{74}}{4608}+\frac{a_{76}}{13824}+\frac{a_{78}}{13824}+\frac{a_{79}}{4608}+\frac{a_{80}}{13824}+\frac{a_{81}}{13824}+\frac{a_{82}}{4608}\\
&+\frac{a_{83}}{13824}+\frac{5a_{84}}{2}+\frac{a_{86}}{2}+\frac{5a_{89}}{6}+\frac{a_{91}}{6}+\frac{5a_{92}}{3}+\frac{a_{93}}{3}+\frac{a_{94}}{3}\\
&+\frac{a_{95}}{6}-\frac{a_{101}}{6}-\frac{5a_{103}}{6}+96a_{104}+\frac{911}{241920}.
\end{aligned} \tag{89}
$$

From the degree 1 part of $\tau_{3,1}\Phi(\tau_{1,0},\tau_{1,0})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0 = &-\frac{7a_2}{144}+\frac{a_3}{96}-\frac{7a_4}{720}+\frac{a_5}{96}-\frac{7a_6}{720}-\frac{7a_7}{1440}-\frac{a_{32}}{144}\\
&-\frac{a_{33}}{96}-\frac{a_{34}}{96}-\frac{a_{35}}{288}-\frac{a_{36}}{288}-\frac{a_{37}}{24}-\frac{a_{38}}{96}-\frac{a_{39}}{96}-\frac{a_{40}}{144}\\
&-\frac{a_{57}}{24}-\frac{a_{58}}{96}-\frac{a_{59}}{96}-\frac{a_{60}}{144}-\frac{a_{61}}{96}-\frac{a_{62}}{96}-\frac{a_{63}}{288}-\frac{a_{64}}{288}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{a_{65}}{144}+\frac{a_{66}}{144}+\frac{a_{67}}{96}+\frac{a_{68}}{96}+\frac{a_{69}}{288}+\frac{a_{70}}{24}+\frac{a_{79}}{1728}+\frac{a_{80}}{6912}\\
&+\frac{a_{81}}{6912}+\frac{a_{83}}{3456}+\frac{10a_{84}}{3}+\frac{2a_{85}}{3}+\frac{2a_{86}}{3}+\frac{a_{87}}{3}+\frac{5a_{92}}{3}+\frac{a_{93}}{3}\\
&+\frac{a_{94}}{3}+\frac{a_{95}}{6}-\frac{a_{100}}{3}-\frac{2a_{101}}{3}-\frac{2a_{102}}{3}-\frac{10a_{103}}{3}+240a_{104}+\frac{5}{6048}.
\end{aligned}
\tag{90}
$$

From the degree 1 part of $\tau_{2,1}\Phi(\tau_{1,0},\tau_{1,1})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{7a_3}{480}+\frac{7a_4}{240}+\frac{a_5}{288}+\frac{7a_7}{480}-\frac{7a_8}{11520}-\frac{7a_{12}}{46080}-\frac{7a_{13}}{46080}-\frac{a_{32}}{144}\\
&-\frac{a_{33}}{288}-\frac{a_{34}}{96}+\frac{a_{35}}{288}-\frac{a_{36}}{288}-\frac{a_{37}}{72}+\frac{a_{38}}{96}-\frac{a_{39}}{288}+\frac{a_{40}}{144}\\
&-\frac{a_{57}}{72}-\frac{a_{59}}{288}-\frac{a_{62}}{288}+\frac{a_{66}}{144}+\frac{a_{67}}{96}+\frac{a_{68}}{96}+\frac{a_{69}}{288}+\frac{a_{70}}{24}\\
&-\frac{a_{72}}{13824}-\frac{a_{74}}{4608}-\frac{a_{76}}{13824}-\frac{a_{78}}{13824}+\frac{5a_{79}}{13824}+\frac{a_{80}}{13824}+\frac{a_{81}}{13824}-\frac{a_{82}}{4608}\\
&+\frac{a_{83}}{4608}+\frac{5a_{84}}{6}+\frac{a_{86}}{6}+\frac{5a_{92}}{8}+\frac{a_{93}}{6}+\frac{a_{94}}{8}+\frac{a_{95}}{12}-\frac{a_{97}}{4}\\
&-\frac{a_{99}}{4}-\frac{a_{100}}{3}-\frac{a_{101}}{2}-\frac{2a_{102}}{3}-\frac{5a_{103}}{2}+144a_{104}-\frac{1}{2304}.
\end{aligned}
\tag{91}
$$

From the degree 1 part of $\tau_{3,0}\Phi(\tau_{1,0},\tau_{1,1})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&-\frac{7a_2}{144}-\frac{43a_3}{720}-\frac{a_4}{10}+\frac{a_5}{72}-\frac{7a_6}{720}-\frac{a_7}{20}+\frac{a_8}{480}\\
&+\frac{a_{12}}{1920}+\frac{a_{13}}{1920}+\frac{7a_{14}}{144}+\frac{7a_{17}}{720}+\frac{7a_{23}}{720}+\frac{7a_{27}}{2880}+\frac{7a_{28}}{2880}+\frac{7a_{30}}{2880}\\
&-\frac{a_{32}}{72}-\frac{5a_{33}}{288}-\frac{a_{34}}{48}-\frac{a_{35}}{144}-\frac{a_{36}}{144}-\frac{5a_{37}}{72}-\frac{a_{38}}{48}-\frac{5a_{39}}{288}\\
&-\frac{a_{40}}{72}+\frac{a_{41}}{288}+\frac{a_{43}}{72}+\frac{a_{45}}{288}+\frac{a_{52}}{72}+\frac{a_{54}}{288}+\frac{a_{56}}{288}-\frac{5a_{57}}{72}\\
&-\frac{a_{58}}{48}-\frac{5a_{59}}{288}-\frac{a_{60}}{72}-\frac{a_{61}}{48}-\frac{5a_{62}}{288}-\frac{a_{63}}{144}-\frac{a_{64}}{144}-\frac{a_{65}}{72}\\
&+\frac{a_{67}}{288}+\frac{a_{70}}{72}+\frac{a_{72}}{6912}+\frac{a_{74}}{2304}+\frac{a_{76}}{6912}+\frac{a_{78}}{6912}+\frac{a_{79}}{768}+\frac{a_{80}}{2304}\\
&+\frac{a_{81}}{1728}+\frac{a_{82}}{1152}+\frac{a_{83}}{1728}+5a_{84}-\frac{2a_{85}}{3}+a_{86}-\frac{a_{87}}{3}+\frac{35a_{92}}{12}\\
&+\frac{7a_{94}}{12}+\frac{a_{97}}{2}+\frac{a_{99}}{2}-\frac{a_{100}}{3}-\frac{2a_{101}}{3}-\frac{2a_{102}}{3}-\frac{10a_{103}}{3}+336a_{104}\\
&-\frac{313}{241920}.
\end{aligned}
\tag{92}
$$

From the degree 1 part of $\tau_{2,1}\Phi(\tau_{1,0},\tau_{2,0})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{a_2}{120}+\frac{3a_5}{160}+\frac{a_6}{96}+\frac{7a_8}{11520}-\frac{7a_9}{138240}-\frac{7a_{10}}{46080}+\frac{7a_{12}}{46080}+\frac{7a_{13}}{46080}\\
&+\frac{a_{32}}{72}-\frac{a_{33}}{48}+\frac{a_{34}}{48}+\frac{a_{36}}{144}-\frac{5a_{37}}{72}-\frac{a_{39}}{72}-\frac{5a_{57}}{72}-\frac{a_{59}}{72}\\
&-\frac{a_{62}}{48}-\frac{a_{66}}{72}-\frac{a_{68}}{48}-\frac{a_{69}}{144}+\frac{a_{70}}{48}+\frac{a_{72}}{13824}+\frac{a_{74}}{4608}-\frac{a_{75}}{13824}\\
&+\frac{a_{76}}{13824}+\frac{a_{78}}{13824}+\frac{a_{79}}{1728}+\frac{a_{81}}{3456}+5a_{84}+a_{86}+\frac{5a_{92}}{2}+\frac{a_{94}}{2}
\end{aligned}
$$

$$- \frac{a_{96}}{2} - \frac{a_{98}}{2} + \frac{2a_{100}}{3} - a_{101} + \frac{4a_{102}}{3} - 5a_{103} + 432a_{104} - \frac{79}{120960}. \tag{93}$$

From the degree 1 part of $\tau_{2,0}\Phi(\tau_{1,0},\tau_{2,1})|_{t=0} = 0$, we obtain

$$\begin{aligned}
0 = &-\frac{a_2}{48} + \frac{7a_3}{480} + \frac{7a_4}{240} + \frac{a_5}{96} - \frac{a_6}{240} + \frac{7a_7}{480} - \frac{7a_8}{11520} \\
&+ \frac{7a_9}{138240} + \frac{7a_{10}}{46080} - \frac{7a_{12}}{46080} - \frac{7a_{13}}{46080} - \frac{a_{32}}{144} - \frac{a_{33}}{288} - \frac{a_{34}}{96} + \frac{a_{35}}{288} \\
&- \frac{a_{36}}{288} - \frac{7a_{37}}{144} + \frac{a_{38}}{96} - \frac{a_{39}}{96} + \frac{a_{40}}{144} - \frac{5a_{57}}{72} + \frac{a_{58}}{96} - \frac{5a_{59}}{288} \\
&+ \frac{a_{60}}{144} - \frac{a_{61}}{96} - \frac{5a_{62}}{288} - \frac{a_{63}}{288} + \frac{a_{64}}{288} - \frac{a_{65}}{144} - \frac{a_{66}}{144} - \frac{a_{67}}{96} \\
&- \frac{a_{68}}{96} - \frac{a_{69}}{288} - \frac{a_{72}}{13824} - \frac{a_{74}}{4608} + \frac{a_{75}}{13824} - \frac{a_{76}}{13824} - \frac{a_{78}}{13824} + \frac{a_{79}}{3456} \\
&+ \frac{a_{80}}{1728} + \frac{a_{82}}{1152} - \frac{a_{83}}{6912} + 5a_{84} + a_{86} + \frac{5a_{92}}{2} + \frac{a_{94}}{2} - \frac{a_{97}}{2} \\
&- \frac{a_{99}}{2} - \frac{a_{101}}{3} - \frac{11a_{103}}{3} + 432a_{104} + \frac{1}{5376}.
\end{aligned} \tag{94}$$

From the degree 1 part of $\tau_{3,1}\Phi(\tau_{1,1},\tau_{0,0})|_{t=0} = 0$, we obtain

$$\begin{aligned}
0 = &-\frac{31a_1}{96768} + \frac{7a_5}{720} + \frac{7a_6}{720} - \frac{7a_9}{34560} - \frac{7a_{10}}{34560} + \frac{7a_{12}}{138240} + \frac{7a_{13}}{138240} \\
&+ \frac{7a_{17}}{1440} - \frac{7a_{21}}{34560} - \frac{7a_{22}}{34560} + \frac{7a_{27}}{2880} + \frac{7a_{28}}{2880} + \frac{7a_{30}}{2880} - \frac{7a_{31}}{138240} - \frac{a_{33}}{288} \\
&- \frac{a_{37}}{144} + \frac{a_{39}}{288} - \frac{a_{41}}{288} - \frac{a_{43}}{144} - \frac{a_{52}}{144} - \frac{a_{54}}{288} - \frac{a_{56}}{288} - \frac{a_{57}}{144} \\
&+ \frac{a_{67}}{288} + \frac{a_{70}}{72} + \frac{a_{72}}{13824} + \frac{a_{74}}{6912} - \frac{a_{75}}{13824} + \frac{a_{76}}{13824} + \frac{a_{78}}{13824} + \frac{a_{79}}{6912} \\
&- \frac{a_{80}}{13824} + \frac{a_{81}}{13824} - \frac{a_{82}}{3456} + \frac{a_{83}}{13824} + \frac{a_{84}}{3} + \frac{a_{89}}{3} + \frac{a_{91}}{12} + \frac{a_{92}}{4} \\
&+ \frac{a_{94}}{24} - \frac{a_{95}}{24} - \frac{a_{98}}{12} - \frac{a_{101}}{6} - \frac{2a_{103}}{3} + 32a_{104} - \frac{a_{105}}{13824}.
\end{aligned} \tag{95}$$

From the degree 1 part of $\tau_{4,0}\Phi(\tau_{1,1},\tau_{0,0})|_{t=0} = 0$, we obtain

$$\begin{aligned}
0 = &\frac{2329a_1}{1451520} - \frac{a_2}{576} - \frac{a_5}{30} - \frac{a_6}{30} - \frac{a_8}{2304} + \frac{a_9}{1440} + \frac{a_{10}}{1440} - \frac{13a_{12}}{46080} \\
&- \frac{13a_{13}}{46080} + \frac{7a_{14}}{576} + \frac{7a_{15}}{576} + \frac{7a_{16}}{576} - \frac{11a_{17}}{720} + \frac{7a_{18}}{576} + \frac{7a_{19}}{576} + \frac{a_{21}}{1440} \\
&+ \frac{a_{22}}{1440} + \frac{a_{23}}{192} + \frac{7a_{24}}{2880} + \frac{7a_{25}}{2880} - \frac{a_{27}}{120} - \frac{a_{28}}{120} + \frac{7a_{29}}{2880} - \frac{a_{30}}{120} \\
&+ \frac{a_{31}}{5760} - \frac{a_{33}}{144} - \frac{7a_{37}}{288} - \frac{a_{39}}{144} - \frac{a_{41}}{144} - \frac{7a_{43}}{288} + \frac{a_{44}}{288} - \frac{a_{45}}{144} \\
&+ \frac{a_{50}}{288} - \frac{7a_{52}}{288} + \frac{a_{53}}{288} - \frac{a_{54}}{144} - \frac{a_{56}}{144} - \frac{7a_{57}}{288} - \frac{a_{59}}{144} - \frac{a_{62}}{144} \\
&+ \frac{a_{70}}{288} + \frac{a_{72}}{13824} + \frac{5a_{74}}{13824} + \frac{a_{75}}{6912} + \frac{a_{76}}{13824} + \frac{a_{78}}{13824} + \frac{5a_{79}}{13824} + \frac{a_{80}}{13824} \\
&+ \frac{a_{81}}{13824} + \frac{a_{82}}{4608} + \frac{a_{83}}{13824} + \frac{3a_{84}}{2} - \frac{a_{86}}{6} + \frac{3a_{89}}{2} + \frac{a_{91}}{6} + \frac{11a_{92}}{12} \\
&+ \frac{a_{95}}{12} + \frac{a_{98}}{6} - \frac{a_{101}}{6} - \frac{5a_{103}}{6} + 72a_{104} + \frac{a_{105}}{6912} - \frac{13}{8064}.
\end{aligned} \tag{96}$$

From the degree 1 part of $\tau_{2,1}\Phi(\tau_{1,1},\tau_{0,1})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0 =& \frac{7a_7}{960}-\frac{7a_{12}}{46080}-\frac{7a_{13}}{46080}-\frac{a_{33}}{288}-\frac{a_{37}}{288}+\frac{a_{40}}{288}-\frac{a_{41}}{576}-\frac{a_{43}}{576}\\
&-\frac{a_{52}}{288}-\frac{a_{57}}{288}-\frac{a_{59}}{576}+\frac{a_{60}}{576}-\frac{a_{62}}{576}+\frac{a_{64}}{576}+\frac{a_{67}}{288}+\frac{a_{70}}{96}\\
&+\frac{a_{72}}{13824}+\frac{a_{74}}{13824}-\frac{a_{76}}{13824}-\frac{a_{78}}{13824}+\frac{a_{79}}{13824}-\frac{a_{80}}{13824}+\frac{a_{81}}{13824}-\frac{a_{82}}{4608}\\
&+\frac{a_{83}}{13824}+\frac{a_{84}}{6}+\frac{a_{89}}{12}+\frac{a_{92}}{8}+\frac{a_{94}}{24}-\frac{a_{99}}{12}-\frac{a_{101}}{6}-\frac{a_{103}}{2}\\
&+24a_{104}.
\end{aligned}\tag{97}
$$

From the degree 1 part of $\tau_{3,0}\Phi(\tau_{1,1},\tau_{0,1})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0 =& -\frac{31a_1}{96768}+\frac{7a_5}{720}+\frac{7a_6}{720}-\frac{a_7}{40}+\frac{7a_8}{34560}-\frac{7a_9}{34560}-\frac{7a_{10}}{34560}\\
&+\frac{a_{12}}{1920}+\frac{a_{13}}{1920}+\frac{7a_{14}}{288}+\frac{7a_{17}}{720}-\frac{7a_{21}}{34560}-\frac{7a_{22}}{34560}+\frac{7a_{23}}{720}+\frac{7a_{27}}{2880}\\
&+\frac{7a_{28}}{2880}+\frac{7a_{30}}{2880}-\frac{7a_{31}}{138240}-\frac{a_{33}}{96}-\frac{a_{37}}{48}+\frac{a_{39}}{288}-\frac{a_{40}}{144}-\frac{a_{41}}{288}\\
&-\frac{a_{43}}{288}+\frac{a_{45}}{288}-\frac{a_{52}}{48}+\frac{a_{54}}{288}+\frac{a_{56}}{288}-\frac{a_{57}}{48}-\frac{a_{59}}{288}-\frac{a_{60}}{288}\\
&-\frac{a_{62}}{288}-\frac{a_{64}}{288}+\frac{a_{67}}{288}+\frac{a_{70}}{72}+\frac{a_{72}}{6912}+\frac{5a_{74}}{13824}-\frac{a_{75}}{13824}+\frac{a_{76}}{6912}\\
&+\frac{a_{78}}{6912}+\frac{5a_{79}}{13824}+\frac{a_{81}}{6912}-\frac{a_{82}}{13824}+\frac{a_{83}}{6912}+\frac{7a_{84}}{6}-\frac{a_{87}}{6}+\frac{7a_{89}}{12}\\
&+\frac{7a_{92}}{8}-\frac{a_{93}}{6}+\frac{a_{94}}{6}-\frac{a_{95}}{8}-\frac{a_{98}}{12}+\frac{a_{99}}{6}-\frac{a_{101}}{3}-\frac{7a_{103}}{6}\\
&+80a_{104}-\frac{a_{105}}{13824}-\frac{31}{48384}.
\end{aligned}\tag{98}
$$

From the degree 1 part of $\tau_{2,1}\Phi(\tau_{1,1},\tau_{1,0})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0 =& \frac{a_3}{288}+\frac{7a_5}{480}+\frac{7a_6}{240}+\frac{7a_7}{480}-\frac{7a_9}{23040}-\frac{7a_{10}}{11520}-\frac{7a_{11}}{46080}-\frac{7a_{13}}{46080}\\
&-\frac{a_{32}}{144}-\frac{a_{33}}{96}-\frac{a_{34}}{288}-\frac{a_{35}}{288}+\frac{a_{36}}{288}-\frac{a_{37}}{72}-\frac{a_{38}}{288}+\frac{a_{39}}{96}\\
&+\frac{a_{40}}{144}-\frac{a_{57}}{72}-\frac{a_{58}}{288}-\frac{a_{61}}{288}+\frac{a_{66}}{144}+\frac{a_{67}}{96}+\frac{a_{68}}{96}+\frac{a_{69}}{288}\\
&+\frac{a_{70}}{24}-\frac{a_{73}}{13824}-\frac{a_{75}}{4608}-\frac{a_{76}}{13824}-\frac{a_{77}}{13824}+\frac{5a_{79}}{13824}+\frac{a_{80}}{13824}+\frac{a_{81}}{13824}\\
&-\frac{a_{82}}{4608}+\frac{a_{83}}{4608}+\frac{5a_{84}}{6}+\frac{a_{85}}{6}+\frac{5a_{92}}{8}+\frac{a_{93}}{8}+\frac{a_{94}}{6}+\frac{a_{95}}{12}\\
&-\frac{a_{98}}{4}-\frac{a_{99}}{4}-\frac{a_{100}}{3}-\frac{2a_{101}}{3}-\frac{a_{102}}{2}-\frac{5a_{103}}{2}+144a_{104}-\frac{1}{4608}.
\end{aligned}\tag{99}
$$

From the degree 1 part of $\tau_{2,0}\Phi(\tau_{1,1},\tau_{1,1})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0 =& \frac{7a_3}{480}+\frac{7a_4}{240}+\frac{7a_5}{480}+\frac{7a_6}{240}+\frac{7a_7}{480}-\frac{7a_8}{11520}-\frac{7a_9}{23040}-\frac{7a_{10}}{11520}\\
&-\frac{7a_{11}}{46080}-\frac{7a_{12}}{46080}-\frac{7a_{13}}{23040}-\frac{a_{33}}{96}-\frac{a_{34}}{96}+\frac{a_{35}}{288}+\frac{a_{36}}{288}-\frac{a_{37}}{48}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{a_{38}}{96}+\frac{a_{39}}{96}+\frac{a_{40}}{144}-\frac{a_{57}}{48}+\frac{a_{60}}{144}+\frac{a_{63}}{288}+\frac{a_{64}}{288}+\frac{a_{67}}{288}\\
&+\frac{a_{68}}{288}+\frac{a_{69}}{288}+\frac{a_{70}}{72}-\frac{a_{72}}{13824}-\frac{a_{73}}{13824}-\frac{a_{74}}{4608}-\frac{a_{75}}{4608}-\frac{a_{76}}{6912}\\
&-\frac{a_{77}}{13824}-\frac{a_{78}}{13824}+\frac{a_{79}}{2304}+\frac{a_{80}}{2304}+\frac{a_{81}}{2304}+\frac{a_{82}}{2304}+\frac{a_{83}}{1728}+a_{84}\\
&+\frac{3a_{92}}{4}+\frac{a_{93}}{8}+\frac{a_{94}}{8}-\frac{a_{97}}{4}-\frac{a_{98}}{4}-\frac{a_{99}}{3}-\frac{5a_{101}}{6}-\frac{5a_{102}}{6}\\
&-\frac{7a_{103}}{3}+192a_{104}-\frac{1}{4608}. \qquad (100)
\end{aligned}
$$

From the degree 1 part of $\tau_{2,0}\Phi(\tau_{1,1},\tau_{2,0})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&-\frac{a_2}{96}-\frac{11a_5}{192}-\frac{379a_6}{2880}+\frac{7a_7}{960}+\frac{7a_8}{11520}+\frac{193a_9}{138240}+\frac{463a_{10}}{138240}\\
&+\frac{7a_{11}}{23040}+\frac{7a_{12}}{46080}+\frac{7a_{13}}{46080}+\frac{7a_{15}}{288}+\frac{7a_{16}}{96}+\frac{7a_{24}}{480}-\frac{a_{32}}{96}-\frac{11a_{33}}{288}\\
&+\frac{a_{34}}{48}-\frac{a_{36}}{144}-\frac{5a_{37}}{72}-\frac{a_{39}}{32}+\frac{a_{40}}{288}+\frac{a_{44}}{48}+\frac{a_{47}}{288}+\frac{a_{53}}{48}\\
&-\frac{13a_{57}}{144}-\frac{a_{59}}{32}+\frac{a_{60}}{288}-\frac{11a_{62}}{288}-\frac{a_{63}}{144}-\frac{a_{65}}{96}-\frac{a_{66}}{96}-\frac{5a_{67}}{288}\\
&-\frac{a_{68}}{144}-\frac{a_{69}}{144}-\frac{a_{70}}{48}+\frac{a_{72}}{13824}+\frac{a_{73}}{6912}+\frac{a_{74}}{4608}+\frac{13a_{75}}{13824}+\frac{a_{76}}{13824}\\
&+\frac{a_{77}}{6912}+\frac{a_{78}}{13824}+\frac{a_{79}}{6912}+\frac{a_{80}}{6912}-\frac{a_{81}}{3456}+\frac{a_{82}}{1152}-\frac{a_{83}}{1728}+\frac{19a_{84}}{3}\\
&-a_{86}+\frac{7a_{92}}{2}-\frac{a_{94}}{6}+\frac{a_{98}}{6}-\frac{a_{99}}{2}-a_{101}+\frac{5a_{102}}{3}-\frac{10a_{103}}{3}\\
&+528a_{104}-\frac{59}{16128}. \qquad (101)
\end{aligned}
$$

From the degree 1 part of $\tau_{3,1}\Phi(\tau_{2,0},\tau_{0,0})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&\frac{31a_1}{96768}-\frac{a_2}{288}+\frac{7a_3}{2880}+\frac{7a_4}{960}-\frac{7a_7}{2880}-\frac{a_8}{1152}+\frac{7a_9}{34560}+\frac{7a_{10}}{34560}\\
&-\frac{7a_{12}}{69120}+\frac{7a_{14}}{288}+\frac{7a_{21}}{34560}+\frac{7a_{22}}{34560}+\frac{a_{23}}{96}+\frac{7a_{31}}{138240}-\frac{a_{32}}{288}+\frac{a_{33}}{144}\\
&-\frac{a_{34}}{288}-\frac{a_{37}}{36}+\frac{a_{38}}{288}-\frac{a_{40}}{288}+\frac{a_{41}}{144}-\frac{a_{43}}{36}-\frac{a_{46}}{288}-\frac{a_{52}}{36}\\
&+\frac{a_{56}}{144}-\frac{a_{57}}{36}+\frac{a_{58}}{288}-\frac{a_{60}}{288}-\frac{a_{61}}{288}-\frac{a_{65}}{288}+\frac{a_{66}}{288}-\frac{a_{67}}{144}\\
&-\frac{a_{68}}{288}+\frac{a_{70}}{48}-\frac{a_{72}}{6912}+\frac{7a_{74}}{13824}+\frac{a_{75}}{13824}-\frac{a_{78}}{6912}+\frac{7a_{79}}{13824}+\frac{a_{80}}{6912}\\
&+\frac{a_{82}}{13824}-\frac{a_{83}}{6912}+\frac{5a_{84}}{3}+\frac{a_{87}}{6}+\frac{5a_{89}}{3}+\frac{5a_{92}}{6}+\frac{a_{95}}{12}-\frac{a_{97}}{6}\\
&-\frac{a_{100}}{6}+\frac{a_{101}}{3}-\frac{5a_{103}}{3}+112a_{104}+\frac{a_{105}}{13824}-\frac{1}{1536}. \qquad (102)
\end{aligned}
$$

From the degree 0 part of $\tau_{2,1}\tau_{2,0}\Phi(\tau_{0,0},\tau_{0,0})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0=&-\frac{31a_1}{10752}-\frac{7a_8}{4608}-\frac{7a_9}{4608}-\frac{7a_{10}}{4608}-\frac{7a_{11}}{23040}-\frac{7a_{12}}{23040}-\frac{7a_{13}}{23040}\\
&-\frac{7a_{20}}{23040}-\frac{7a_{21}}{4608}-\frac{7a_{22}}{4608}-\frac{7a_{31}}{23040}-\frac{a_{71}}{6912}-\frac{a_{72}}{6912}-\frac{a_{73}}{6912}-\frac{a_{74}}{2304}
\end{aligned}
$$

$$- \frac{a_{75}}{2304} - \frac{a_{76}}{6912} - \frac{a_{77}}{6912} - \frac{a_{78}}{6912} - \frac{a_{79}}{2304} - \frac{a_{80}}{6912} - \frac{a_{81}}{6912} - \frac{a_{82}}{2304}$$
$$- \frac{a_{83}}{6912} - \frac{a_{105}}{2304} + \frac{31}{1536}. \tag{103}$$

From the degree 0 part of $\tau_{2,0}\tau_{3,0}\Phi(\tau_{0,0},\tau_{0,0})|_{t=0} = 0$, we obtain

$$\begin{aligned}
0 = {} & \frac{859a_1}{48384} + \frac{7a_2}{48} + \frac{7a_3}{720} + \frac{7a_4}{720} + \frac{7a_5}{720} + \frac{7a_6}{720} + \frac{7a_7}{960} + \frac{229a_8}{34560} \\
& + \frac{229a_9}{34560} + \frac{229a_{10}}{34560} + \frac{19a_{11}}{23040} + \frac{19a_{12}}{23040} + \frac{a_{13}}{1920} + \frac{7a_{14}}{48} + \frac{7a_{15}}{48} + \frac{7a_{16}}{48} \\
& + \frac{7a_{17}}{720} + \frac{7a_{18}}{48} + \frac{7a_{19}}{48} + \frac{19a_{20}}{23040} + \frac{229a_{21}}{34560} + \frac{229a_{22}}{34560} + \frac{7a_{23}}{288} + \frac{7a_{24}}{288} \\
& + \frac{7a_{25}}{288} + \frac{7a_{26}}{2880} + \frac{7a_{27}}{2880} + \frac{7a_{28}}{2880} + \frac{7a_{29}}{288} + \frac{7a_{30}}{2880} + \frac{31a_{31}}{23040} + \frac{a_{32}}{288} \\
& + \frac{a_{33}}{288} + \frac{a_{34}}{288} + \frac{5a_{37}}{144} + \frac{a_{38}}{288} + \frac{a_{39}}{288} + \frac{a_{40}}{288} + \frac{a_{41}}{288} + \frac{a_{42}}{288} \\
& + \frac{5a_{43}}{144} + \frac{5a_{44}}{144} + \frac{a_{45}}{288} + \frac{a_{46}}{288} + \frac{a_{47}}{288} + \frac{a_{49}}{288} + \frac{5a_{50}}{144} + \frac{a_{51}}{288} \\
& + \frac{5a_{52}}{144} + \frac{5a_{53}}{144} + \frac{a_{54}}{288} + \frac{a_{55}}{288} + \frac{a_{56}}{288} + \frac{5a_{57}}{144} + \frac{a_{58}}{288} + \frac{a_{59}}{288} \\
& + \frac{a_{60}}{288} + \frac{a_{61}}{288} + \frac{a_{62}}{288} + \frac{a_{65}}{288} + \frac{a_{66}}{288} + \frac{a_{67}}{288} + \frac{a_{68}}{288} + \frac{5a_{70}}{144} \\
& + \frac{a_{71}}{3456} + \frac{a_{72}}{3456} + \frac{a_{73}}{3456} + \frac{a_{74}}{768} + \frac{a_{75}}{768} + \frac{a_{76}}{6912} + \frac{a_{77}}{3456} + \frac{a_{78}}{3456} \\
& + \frac{a_{79}}{768} + \frac{a_{80}}{6912} + \frac{a_{81}}{6912} + \frac{a_{82}}{2304} + \frac{a_{83}}{3456} + \frac{a_{105}}{768} - \frac{859}{6912}.
\end{aligned} \tag{104}$$

From the degree 1 part of $\tau_{3,1}\tau_{3,0}\Phi(\tau_{0,0},\tau_{0,0})|_{t=0} = 0$, we obtain

$$\begin{aligned}
0 = {} & \frac{53a_1}{48384} + \frac{a_2}{48} + \frac{7a_3}{720} + \frac{7a_4}{720} + \frac{7a_5}{720} + \frac{7a_6}{720} - \frac{79a_7}{2880} - \frac{61a_8}{34560} \\
& - \frac{61a_9}{34560} - \frac{61a_{10}}{34560} - \frac{43a_{11}}{69120} - \frac{43a_{12}}{69120} + \frac{a_{13}}{1920} + \frac{a_{14}}{12} + \frac{7a_{15}}{48} + \frac{a_{16}}{12} \\
& + \frac{7a_{17}}{720} + \frac{a_{18}}{12} + \frac{7a_{19}}{48} - \frac{43a_{20}}{69120} - \frac{61a_{21}}{34560} - \frac{61a_{22}}{34560} + \frac{7a_{23}}{288} + \frac{7a_{24}}{288} \\
& + \frac{7a_{25}}{288} + \frac{7a_{26}}{2880} + \frac{7a_{27}}{2880} + \frac{7a_{28}}{2880} + \frac{7a_{29}}{288} + \frac{7a_{30}}{2880} - \frac{23a_{31}}{69120} - \frac{a_{32}}{96} \\
& + \frac{a_{33}}{288} + \frac{a_{34}}{288} - \frac{a_{37}}{16} + \frac{a_{38}}{288} + \frac{a_{39}}{288} - \frac{a_{40}}{96} + \frac{a_{41}}{288} + \frac{a_{42}}{288} \\
& - \frac{a_{43}}{16} - \frac{a_{44}}{16} + \frac{a_{45}}{288} - \frac{a_{46}}{288} - \frac{a_{47}}{288} - \frac{a_{49}}{288} - \frac{a_{50}}{16} + \frac{a_{51}}{288} \\
& - \frac{a_{52}}{16} - \frac{a_{53}}{16} + \frac{a_{54}}{288} + \frac{a_{55}}{288} + \frac{a_{56}}{288} - \frac{a_{57}}{16} + \frac{a_{58}}{288} + \frac{a_{59}}{288} \\
& - \frac{a_{60}}{96} - \frac{a_{61}}{288} - \frac{a_{62}}{288} - \frac{a_{65}}{96} + \frac{a_{66}}{288} - \frac{a_{67}}{96} - \frac{a_{68}}{96} + \frac{5a_{70}}{144} \\
& - \frac{a_{71}}{3456} - \frac{a_{72}}{3456} - \frac{a_{73}}{3456} + \frac{a_{74}}{768} + \frac{a_{75}}{768} + \frac{a_{76}}{6912} - \frac{a_{77}}{3456} - \frac{a_{78}}{3456} \\
& + \frac{a_{79}}{768} + \frac{a_{80}}{6912} + \frac{a_{81}}{6912} + \frac{a_{82}}{2304} - \frac{a_{83}}{3456} + \frac{11a_{84}}{3} + \frac{11a_{88}}{3} + \frac{11a_{89}}{3} \\
& + \frac{11a_{90}}{3} + \frac{23a_{92}}{12} + \frac{a_{95}}{12} - \frac{a_{97}}{6} - \frac{a_{98}}{6} + \frac{a_{99}}{6} - \frac{a_{100}}{3} + \frac{a_{101}}{3}
\end{aligned}$$

$$+\frac{a_{102}}{3}-\frac{10a_{103}}{3}+240a_{104}+\frac{a_{105}}{768}-\frac{53}{6912}. \tag{105}$$

From the degree 1 part of $\tau_{2,1}\tau_{3,1}\Phi(\tau_{0,0},\tau_{0,0})|_{t=0}=0$, we obtain

$$\begin{aligned}
0 = & -\frac{5a_1}{48384}+\frac{a_2}{144}+\frac{7a_3}{720}+\frac{7a_4}{720}+\frac{7a_5}{720}+\frac{7a_6}{720}+\frac{7a_7}{960} \\
& -\frac{5a_8}{13824}-\frac{5a_9}{13824}-\frac{5a_{10}}{13824}+\frac{7a_{11}}{34560}+\frac{7a_{12}}{34560}-\frac{7a_{13}}{69120}+\frac{a_{14}}{72}+\frac{a_{15}}{48} \\
& +\frac{a_{16}}{72}+\frac{7a_{17}}{1440}+\frac{a_{18}}{72}+\frac{a_{19}}{48}+\frac{7a_{20}}{34560}-\frac{5a_{21}}{13824}-\frac{5a_{22}}{13824}+\frac{a_{23}}{288} \\
& +\frac{a_{24}}{288}+\frac{a_{25}}{288}+\frac{7a_{26}}{2880}+\frac{7a_{27}}{2880}+\frac{7a_{28}}{2880}+\frac{a_{29}}{288}+\frac{7a_{30}}{2880}-\frac{17a_{31}}{138240} \\
& -\frac{a_{32}}{288}-\frac{a_{33}}{288}-\frac{a_{34}}{288}-\frac{a_{37}}{72}+\frac{a_{38}}{288}+\frac{a_{39}}{288}+\frac{a_{40}}{288}-\frac{a_{41}}{288} \\
& -\frac{a_{42}}{288}-\frac{a_{43}}{72}-\frac{a_{44}}{72}-\frac{a_{46}}{288}-\frac{a_{47}}{288}-\frac{a_{49}}{288}-\frac{a_{50}}{72}-\frac{a_{51}}{288} \\
& -\frac{a_{52}}{72}-\frac{a_{53}}{72}-\frac{a_{54}}{288}-\frac{a_{55}}{288}-\frac{a_{56}}{288}-\frac{a_{57}}{72}+\frac{a_{66}}{288}+\frac{a_{67}}{288} \\
& +\frac{a_{68}}{288}+\frac{5a_{70}}{144}+\frac{a_{71}}{6912}+\frac{a_{72}}{6912}+\frac{a_{73}}{6912}+\frac{5a_{74}}{13824}+\frac{5a_{75}}{13824}+\frac{a_{77}}{6912} \\
& +\frac{a_{78}}{6912}+\frac{5a_{79}}{13824}-\frac{7a_{82}}{13824}+\frac{a_{83}}{6912}+\frac{2a_{84}}{3}+\frac{2a_{88}}{3}+\frac{2a_{89}}{3}+\frac{2a_{90}}{3} \\
& +\frac{a_{91}}{12}+\frac{a_{92}}{2}+\frac{a_{93}}{24}+\frac{a_{94}}{24}-\frac{a_{97}}{12}-\frac{a_{98}}{12}-\frac{a_{99}}{12}-\frac{a_{100}}{6} \\
& -\frac{a_{101}}{6}-\frac{a_{102}}{6}-\frac{5a_{103}}{3}+80a_{104}+\frac{5a_{105}}{13824}-\frac{7}{4608}.
\end{aligned} \tag{106}$$

From the degree 1 part of $\tau_{2,1}\tau_{4,0}\Phi(\tau_{0,0},\tau_{0,0})|_{t=0}=0$, we obtain

$$\begin{aligned}
0 = & \frac{923a_1}{290304}+\frac{17a_2}{576}-\frac{a_3}{30}-\frac{a_4}{30}-\frac{a_5}{30}-\frac{a_6}{30}-\frac{109a_8}{69120}-\frac{109a_9}{69120} \\
& -\frac{109a_{10}}{69120}-\frac{a_{11}}{7680}-\frac{a_{12}}{7680}-\frac{a_{13}}{7680}+\frac{5a_{14}}{64}+\frac{73a_{15}}{576}+\frac{5a_{16}}{64}-\frac{11a_{17}}{720} \\
& +\frac{5a_{18}}{64}+\frac{73a_{19}}{576}-\frac{a_{20}}{7680}-\frac{109a_{21}}{69120}-\frac{109a_{22}}{69120}+\frac{23a_{23}}{960}+\frac{23a_{24}}{960}+\frac{23a_{25}}{960} \\
& -\frac{a_{26}}{120}-\frac{a_{27}}{120}-\frac{a_{28}}{120}+\frac{23a_{29}}{960}-\frac{a_{30}}{120}-\frac{7a_{31}}{46080}+\frac{a_{32}}{144}-\frac{a_{33}}{144} \\
& -\frac{a_{34}}{144}-\frac{17a_{37}}{288}-\frac{a_{38}}{144}-\frac{a_{39}}{144}-\frac{a_{41}}{144}-\frac{a_{42}}{144}-\frac{17a_{43}}{288}-\frac{17a_{44}}{288} \\
& -\frac{a_{45}}{144}-\frac{17a_{50}}{288}-\frac{a_{51}}{144}-\frac{17a_{52}}{288}-\frac{17a_{53}}{288}-\frac{a_{54}}{144}-\frac{a_{55}}{144}-\frac{a_{56}}{144} \\
& -\frac{17a_{57}}{288}-\frac{a_{58}}{144}-\frac{a_{59}}{144}-\frac{a_{61}}{144}-\frac{a_{62}}{144}-\frac{a_{66}}{144}+\frac{a_{70}}{96}+\frac{a_{71}}{6912} \\
& +\frac{a_{72}}{6912}+\frac{a_{73}}{6912}+\frac{11a_{74}}{13824}+\frac{11a_{75}}{13824}+\frac{a_{76}}{6912}+\frac{a_{77}}{6912}+\frac{a_{78}}{6912}+\frac{11a_{79}}{13824} \\
& +\frac{a_{80}}{6912}+\frac{a_{81}}{6912}+\frac{5a_{82}}{13824}+\frac{a_{83}}{6912}+\frac{7a_{84}}{2}-\frac{a_{85}}{6}-\frac{a_{86}}{6}+\frac{7a_{88}}{2} \\
& +\frac{7a_{89}}{2}+\frac{7a_{90}}{2}+\frac{a_{91}}{6}+2a_{92}+\frac{a_{95}}{12}-\frac{a_{96}}{6}+\frac{a_{97}}{6}+\frac{a_{98}}{6} \\
& +\frac{a_{100}}{3}-\frac{a_{101}}{6}-\frac{a_{102}}{6}-\frac{5a_{103}}{2}+200a_{104}+\frac{11a_{105}}{13824}-\frac{5489}{483840}.
\end{aligned} \tag{107}$$

From the degree 1 part of $\tau_{2,0}\tau_{4,1}\Phi(\tau_{0,0},\tau_{0,0})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0 = & \frac{139a_1}{241920} + \frac{a_2}{240} + \frac{a_3}{240} - \frac{a_4}{720} + \frac{a_5}{240} - \frac{a_6}{720} + \frac{7a_7}{960} - \frac{7a_8}{4608} \\
& - \frac{89a_9}{69120} - \frac{7a_{10}}{4608} + \frac{a_{11}}{11520} + \frac{a_{12}}{11520} - \frac{a_{13}}{4608} + \frac{11a_{14}}{240} + \frac{7a_{15}}{80} + \frac{11a_{16}}{240} \\
& + \frac{a_{17}}{720} + \frac{11a_{18}}{240} + \frac{7a_{19}}{80} + \frac{a_{20}}{11520} - \frac{89a_{21}}{69120} - \frac{7a_{22}}{4608} + \frac{7a_{23}}{480} + \frac{7a_{24}}{480} \\
& + \frac{7a_{25}}{480} + \frac{a_{26}}{960} + \frac{a_{27}}{960} + \frac{a_{28}}{960} + \frac{7a_{29}}{480} + \frac{a_{30}}{960} - \frac{11a_{31}}{46080} - \frac{a_{32}}{288} \\
& - \frac{a_{33}}{288} - \frac{a_{34}}{288} - \frac{5a_{37}}{144} - \frac{a_{38}}{288} - \frac{a_{39}}{288} + \frac{a_{40}}{288} - \frac{a_{41}}{288} - \frac{a_{42}}{288} \\
& - \frac{5a_{43}}{144} - \frac{5a_{44}}{144} - \frac{a_{45}}{288} - \frac{a_{46}}{288} - \frac{a_{47}}{288} - \frac{a_{49}}{288} - \frac{5a_{50}}{144} - \frac{a_{51}}{288} \\
& - \frac{5a_{52}}{144} - \frac{5a_{53}}{144} - \frac{a_{54}}{288} - \frac{a_{55}}{288} - \frac{a_{56}}{288} - \frac{a_{57}}{24} - \frac{a_{58}}{288} - \frac{a_{59}}{288} \\
& + \frac{a_{60}}{288} - \frac{a_{61}}{288} - \frac{a_{62}}{288} - \frac{a_{65}}{288} - \frac{a_{66}}{288} + \frac{a_{67}}{288} + \frac{a_{68}}{288} + \frac{a_{70}}{48} \\
& + \frac{a_{71}}{6912} + \frac{a_{72}}{6912} + \frac{a_{73}}{6912} + \frac{5a_{74}}{13824} + \frac{5a_{75}}{13824} + \frac{a_{77}}{6912} + \frac{a_{78}}{6912} + \frac{5a_{79}}{13824} \\
& + \frac{a_{82}}{13824} + \frac{a_{83}}{6912} + \frac{5a_{84}}{2} + \frac{a_{85}}{6} + \frac{a_{86}}{6} + \frac{5a_{88}}{2} + \frac{5a_{89}}{2} + \frac{5a_{90}}{2} \\
& + \frac{a_{91}}{6} + \frac{5a_{92}}{4} + \frac{a_{93}}{12} + \frac{a_{94}}{12} - \frac{a_{99}}{6} - \frac{a_{101}}{6} - \frac{a_{102}}{6} - \frac{13a_{103}}{6} \\
& + 160a_{104} + \frac{5a_{105}}{13824} - \frac{199}{53760}.
\end{aligned} \tag{108}
$$

From the degree 1 part of $\tau_{2,1}\tau_{2,1}\Phi(\tau_{0,0},\tau_{0,1})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0 = & \frac{a_1}{13824} + \frac{a_2}{288} + \frac{7a_7}{480} - \frac{a_8}{13824} - \frac{a_9}{13824} - \frac{5a_{10}}{13824} + \frac{7a_{11}}{23040} - \frac{7a_{12}}{23040} \\
& - \frac{7a_{13}}{23040} + \frac{a_{14}}{576} + \frac{a_{15}}{288} + \frac{7a_{16}}{576} + \frac{7a_{18}}{576} + \frac{a_{19}}{288} + \frac{7a_{20}}{23040} - \frac{a_{21}}{13824} \\
& - \frac{5a_{22}}{13824} + \frac{a_{24}}{288} + \frac{a_{25}}{288} + \frac{a_{29}}{576} - \frac{a_{31}}{13824} - \frac{a_{33}}{144} - \frac{a_{37}}{144} + \frac{a_{40}}{144} \\
& - \frac{a_{41}}{288} - \frac{a_{43}}{288} - \frac{a_{44}}{144} - \frac{a_{47}}{288} - \frac{a_{49}}{288} - \frac{a_{50}}{144} - \frac{a_{52}}{144} - \frac{a_{53}}{144} \\
& - \frac{a_{57}}{144} - \frac{a_{59}}{288} + \frac{a_{60}}{288} - \frac{a_{62}}{288} + \frac{a_{64}}{288} + \frac{a_{67}}{144} + \frac{a_{70}}{48} + \frac{a_{71}}{6912} \\
& + \frac{a_{72}}{6912} + \frac{a_{73}}{6912} + \frac{a_{74}}{6912} + \frac{a_{75}}{6912} - \frac{a_{76}}{6912} + \frac{a_{77}}{6912} - \frac{a_{78}}{6912} + \frac{a_{79}}{6912} \\
& - \frac{a_{80}}{6912} + \frac{a_{81}}{6912} - \frac{a_{82}}{2304} + \frac{a_{83}}{6912} + \frac{a_{84}}{3} + \frac{a_{88}}{3} + \frac{a_{89}}{6} + \frac{a_{90}}{3} \\
& + \frac{a_{92}}{4} + \frac{a_{94}}{12} - \frac{a_{99}}{6} - \frac{a_{101}}{3} - a_{103} + 48a_{104} + \frac{a_{105}}{6912} - \frac{1}{4608}.
\end{aligned} \tag{109}
$$

From the degree 1 part of $\tau_{2,1}\tau_{2,1}\Phi(\tau_{0,0},\tau_{1,0})|_{t=0}=0$, we obtain

$$
\begin{aligned}
0 = & \frac{a_2}{48} + \frac{5a_3}{288} + \frac{a_4}{288} + \frac{7a_5}{240} + \frac{7a_6}{120} + \frac{7a_7}{160} - \frac{5a_9}{6912} - \frac{5a_{10}}{3456} \\
& + \frac{a_{11}}{69120} - \frac{41a_{13}}{69120} + \frac{11a_{15}}{288} + \frac{a_{16}}{16} + \frac{a_{17}}{96} + \frac{5a_{24}}{288} + \frac{a_{28}}{288} + \frac{a_{30}}{288}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{a_{32}}{48}-\frac{a_{33}}{48}-\frac{a_{34}}{144}-\frac{a_{35}}{144}+\frac{a_{36}}{144}-\frac{5a_{37}}{144}-\frac{a_{38}}{144}+\frac{a_{39}}{48}\\
&+\frac{a_{40}}{48}-\frac{a_{42}}{144}-\frac{5a_{44}}{144}-\frac{a_{45}}{144}-\frac{a_{47}}{48}-\frac{a_{48}}{144}-\frac{5a_{53}}{144}-\frac{a_{54}}{144}\\
&-\frac{a_{55}}{144}-\frac{5a_{57}}{144}-\frac{a_{58}}{144}-\frac{a_{61}}{144}+\frac{a_{66}}{48}+\frac{a_{67}}{48}+\frac{a_{68}}{48}+\frac{a_{69}}{144}\\
&+\frac{5a_{70}}{48}+\frac{a_{73}}{2304}+\frac{7a_{75}}{6912}+\frac{a_{76}}{6912}+\frac{a_{77}}{2304}+\frac{7a_{79}}{6912}+\frac{a_{80}}{6912}+\frac{a_{81}}{6912}\\
&-\frac{a_{82}}{2304}+\frac{a_{83}}{2304}+2a_{84}+\frac{a_{85}}{3}+2a_{90}+\frac{a_{91}}{3}+\frac{3a_{92}}{2}+\frac{a_{93}}{4}\\
&+\frac{a_{94}}{3}+\frac{a_{95}}{4}-\frac{a_{98}}{2}-\frac{2a_{99}}{3}-a_{100}-\frac{4a_{101}}{3}-a_{102}-6a_{103}\\
&+336a_{104}-\frac{25}{6912}. \qquad (110)
\end{aligned}
$$

References

[AC] Arbarello, E., and Cornalba, M., *Calculating cohomology groups of moduli spaces of curves via algebraic geometry*, Inst. Hautes Études Sci. Publ. Math. No. 88 (1998), 97–127.

[BP] Belorousski, P. and Pandharipande, R., *A descendent relation in genus 2*, Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) 29 (2000) 171-191.

[DZ] Dubrovin, B., Zhang, Y., *Bihamiltonian hierarchies in 2D topological field theory at one-loop approximation*, Comm. Math. Phys. 198 (1998) 311 - 361.

[EHX] Eguchi, T., Hori, K., and Xiong, C., *Quantum Cohomology and Virasoro Algebra*, Phys. Lett. B402 (1997) 71-80.

[FP1] Faber, C., and Pandharipande, R., *Logarithmic series and Hodge integrals in the tautological ring*, Michigan Math. J. 48 (2000), 215-252.

[FP2] Faber, C., and Pandharipande, R., *Relative maps and tautological classes*, J. Eur. Math. Soc. (JEMS) 7 (2005), no. 1, 13-49.

[Ga] Gathmann, A., *Topological recursion relations and Gromov-Witten invariants in higher genus*, (math.AG/0305361).

[Ge1] Getzler, E., *Intersection theory on $\bar{M}_{1,4}$ and elliptic Gromov-Witten Invariants*, J. Amer. Math. Soc. 10 (1997) 973-998

[Ge2] Getzler, E., *Topological recursion relations in genus 2*, Integrable systems and algebraic geometry (Kobe/Kyoto, 1997), 73-106.

[Gi] Givental, A., *Gromov-Witten invariants and quantization of quadratic hamiltonians*, Moscow Mathematical Journal, v.1, no. 4 (2001), 551-568.

[Io] Ionel, E. *Topological recursive relations in $H^{2g}(\mathcal{M}_{g,n})$*. Invent. Math. 148 (2002), no. 3, 627–658.

[K] Kontsevich, M., *Intersection theory on the moduli space of curves and the matrix airy function*, Comm. Math. Phys., 147 (1992).

[KL] Kimura, T. and Liu, X., *A genus 3 topological recursion relation*, Comm. Math. Phys. 262 (2006), no. 3, 645-661.

[LiT] Li, J. and Tian, G., *Virtual moduli cycles and Gromov-Witten invariants of general symplectic manifolds*, Topics in symplectic 4-manifolds (Irvine, CA, 1996), 47-83.

[L1] Liu, X., *Quantum product on the big phase space and Virasoro conjecture*, Advances in Mathematics 169 (2002), 313-375.

[L2] Liu, X., *Relations among universal equations for Gromov-Witten invariants*, C. Hertling and M. Marcolli (Eds), Aspects of Mathematics, A publication of Max-Planck-Institute for mathematics, Bonn, pp 169 - 180, 2004. (*math.DG/0301161*)

[LP] Liu, X. and Pandharipande, R., *New topological recursion relations*, to appear in J. Alg. Geom., (*math.AG/0805.4829*).

[P] Pandharipande, R., *Three questions in Gromov-Witten theory.* Proceedings of the ICM 2002 Beijing, vol. II, 503-512.

[RT] Ruan, Y. and Tian, G., *Higher genus symplectic invariants and sigma models coupled with gravity*, Invent. Math. 130 (1997), 455-516.

[W] Witten, E., *Two dimensional gravity and intersection theory on Moduli space*, Surveys in Diff. Geom., 1 (1991), 243-310.

[Y] Yang, S., *Calculating intersection numbers on moduli spaces of curves*, *math.AG/08081974*

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